# Errata and comments for the book:

## String Theory in a Nutshell

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# 1 Chapter 1

# 2 Chapter 2

## Section 2.2: Relativistic strings

- In equation (2.2.15) the last sign should be reverted to agree with the definition

$$T_{\alpha\beta} \equiv -\frac{4\pi}{\sqrt{-\det g}} \frac{\delta S_P}{\delta g^{\alpha\beta}} = \frac{1}{\ell_s^2}\left[\partial_\alpha X \cdot \partial_\beta X - \frac{1}{2} g_{\alpha\beta} g^{\gamma\delta} \partial_\gamma X \cdot \partial_\delta X\right]. \tag{2.0.1}$$

Thanks to Richard Garavuso for bringing this to my attention

- Equation (2.2.34) should read

$$\delta S = -4T \int d^2\xi (\delta X^\mu \partial_+ \partial_- X_\mu) - T \int d\tau \left[ X'_\mu \delta X^\mu \Big|_{\sigma=0} - X'_\mu \delta X^\mu \Big|_{\sigma=\bar{\sigma}} \right]. \tag{2.0.2}$$

Thanks to Moritz McGarrie for bringing this to my attention.

- Equations (2.2.38) and (2.2.40) should read respectively

$$T_{10} = T_{01} = \frac{1}{\ell_s^2} \dot{X} \cdot X' = 0 \quad , \quad T_{00} = T_{11} = \frac{1}{2\ell_s^2}(\dot{X}^2 + X'^2) = 0, \tag{2.0.3}$$

and

$$T_{++} = \frac{1}{\ell_s^2} \partial_+ X \cdot \partial_+ X \quad , \quad T_{--} = \frac{1}{\ell_s^2} \partial_- X \cdot \partial_- X \quad , \quad T_{+-} = T_{-+} = 0. \tag{2.0.4}$$

in order to agree with the definition in (2.2.15).

Thanks to Richard Garavuso for bringing this to my attention

## Section 2.3.2: Open strings

Equation (2.3.28) should read as

$$\text{DD} \quad : \quad X^I(\tau,\sigma) = x^I + w^I \sigma + \sqrt{2}\ell_s \sum_{n \in \mathbb{Z}-\{0\}} \frac{\alpha^I{}_n}{n} e^{-in\tau} \sin(n\sigma). \tag{2.0.1}$$

Thanks to Jose-Ignacio Rosado-Sanchez for bringing this to my attention.

# 3 Chapter 3

## Section 3.1: Covariant Canonical Quantization

- Equations (3.1.11) to (3.1.12) should be replaced by

$$\frac{a_0 \cdot a_0}{2} - 1 + N = 0 \tag{3.0.1}$$

where $N$ is level-number operator

$$N = \sum_{m=1}^{\infty} a_{-m} \cdot a_m \tag{3.0.2}$$

For open strings $a_0^\mu = \sqrt{2}\ell_s p^\mu$ and the mass-shell condition becomes

$$\ell_s^2 m^2 = N - 1 \tag{3.0.3}$$

For closed strings, $a_0^\mu = \frac{\ell_s p^\mu}{\sqrt{2}}$ and the mass shell condition becomes

$$\ell_s^2 m^2 = 4(N - 1) \tag{3.0.4}$$

We can deduce a similar expression .....

Thanks to Eduard Balzin for bringing this to my attention.

## Section 3.4.1: Open strings and Chan-Paton factors

- In equation (3.4.8) the order of $\gamma_\Omega$ and $\gamma_\Omega^{-1}$ should be interchanged so that it is compatible with later equations:

$$\Omega \ : \ |p\,;ij\rangle \to \epsilon\ (\gamma_\Omega^{-1})_{ii'}\ |p\,;,j'i'\rangle\ (\gamma_\Omega)_{j'j}\ ,$$

Thanks to Jose Rosado Sanchez for bringing this to my attention.

- The text between equations (3.4.10) and (3.4.15) should read

Unless $\zeta$ is a sign, $\zeta^2 = 1$, the only solution to (3.4.10) is $\gamma_\Omega = 0$. Taking the determinant of (3.4.10) we obtain

$$\zeta^{\mathrm{N}} = 1\ . \tag{3.0.1}$$

Therefore both $\epsilon$ and $\zeta$ are signs.

We will now derive further constraints on $\epsilon$ and $\zeta$, and eventually classify the solutions of (3.4.10). The strongest constraints can be obtained by considering the massless vector states

$$A^\mu = \alpha_{-1}^\mu \sum_{ij} |p\,;ij\rangle\ \lambda_{ij}\ . \tag{3.0.2}$$

The vectors that will survive the $\Omega$ projection have eigenvalue 1. Their CP wave-functions $\lambda$ must satisfy

$$\lambda = -\epsilon \; \gamma_\Omega \; \lambda^T \; \gamma_\Omega^{-1} \; , \tag{3.0.3}$$

where we used (3.4.5). The gauge group is the space of solutions of (3.4.13) where $\lambda$ is hermitian.

So far, such constraints on the phases are true for any state of the open string. However, for the vectors there is a further constraint: that their CP matrices form a Lie algebra. This is necessary for the consistency of their interactions. Using, (3.4.13) we obtain for the commutator of two matrices

## Section 3.5: Path integral quantization

- Equation (3.5.1) on page 37 should be replaced by:

$$Z = \int \frac{\mathcal{D}g\mathcal{D}X^\mu}{V_{\text{gauge}}} \; e^{iS_p(g,X^\mu)}$$

where we used Minkowski signature for consistency with the rest of this section.

- Equation (3.5.6) on page 38 should be replaced by:

$$\sqrt{\det \hat{P}\hat{P}^\dagger} = \int \mathcal{D}c\mathcal{D}b \; e^{-\frac{i}{2\pi}\int d^2\xi \sqrt{\hat{h}} \; \hat{h}^{\alpha\beta} b_{\alpha\gamma} \nabla_\beta c^\gamma} \, .$$

## Section 3.7: BRST primer

- Equation (3.7.6) should read

$$\delta(b_A F^A) = i\epsilon[iB_A F^A(\phi) + b_A c^\alpha \delta_\alpha F^A(\phi)] = i\epsilon(S_1 + S_2) \, . \tag{3.0.1}$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 3.8: BRST in string theory and the physical spectrum

- Equation (3.8.1) on page 42 should become:

$$\begin{aligned} \delta_B X^\mu &= i\epsilon(c^+\partial_+ + c^-\partial_-)X^\mu\,, \\ \delta_B c^\pm &= i\epsilon(c^+\partial_+ + c^-\partial_-)c^\pm\,, \\ \delta_B b_\pm &= i\epsilon(T^X_\pm + T^{gh}_\pm)\,. \end{aligned} \tag{3.0.1}$$

The following explanation may also helpful: The variation for the $b$ ghosts is different from (3.7.5) because we integrated out the antighost $B$. Indeed, this field implements the gauge fixing condition via $S_B = -\frac{1}{4\pi}\int B_{++}(g^{++}-h^{++}) + B_{--}(g^{--}-h^{--})$. The stress tensors now are modified because of this part by $T_{\pm\pm} \to T_{\pm\pm} - B_{\pm\pm}$ The $g^{\pm\pm}$ equations now imply $B_{\pm\pm} = T_{\pm\pm}$ and this explains the variation of the $b$ ghosts above.

- Equation (3.8.3) on page 42 should become:

$$T^{gh}_{++} = (2b_{++}\partial_+ c^+ + \partial_+ b_{++} c^+) \quad , \quad T^{gh}_{--} = (2b_{--}\partial_- c^- + \partial_- b_{--} c^-)\,,$$

- Equation (3.8.6) on page 43 should become:

$$\begin{aligned} c^+ &= \sum c_n\ e^{-in(\tau+\sigma)}\,, \quad c^- = \sum \bar{c}_n\ e^{-in(\tau-\sigma)}\,, \\ b_{++} &= \sum b_n\ e^{-in(\tau+\sigma)}\,, \quad b_{--} = \sum \bar{b}_n\ e^{-in(\tau-\sigma)}\,. \end{aligned}$$

- Equation (3.8.19) on page 45 should become:

$$Q_B = Q_0 + Q_1 + \cdots \quad , \quad Q_0 = c_0(L^X_0 - 1) \quad , \quad Q_1 = c_1 L^X_{-1} + c_{-1}L^X_1 + c_0(c_{-1}b_1 + b_{-1}c_1)$$

# 4 Chapter 4

## Section 4.1: Conformal transformations

- On page 50 and after equation 4.1.7 the phrase "Indeed for $d > 2$, (4.1.7) implies that the parameter $\epsilon$ can be at most quadratic in $x$. " This is because (4.1.7) is cubic in derivatives and non-degenerate.

  should be changed to:

  Indeed for $d > 2$, (4.1.7) implies that the parameter $\partial \cdot \epsilon$ can be at most linear in $x$". This is because the operator $\delta_{\mu\nu}\Box + (d-2)\partial_\mu\partial_\nu$ is non-degenerate.

  Thanks to Lorenzo Battarra for bringing this to my attention.

- Equation (4.1.11) should read

  $$P_\mu = -i\partial_\mu \quad , \quad J_{\mu\nu} = i(x_\mu\partial_\nu - x_\nu\partial_\mu) \quad , \quad K_\mu = -i\left[x^2\partial_\mu - 2x_\mu(x\cdot\partial)\right] \quad , \quad D = ix\cdot\partial \tag{4.0.1}$$

  in order to be consistent with the commutation relations.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.2: Conformally invariant field theory

- On page 52, in equations 4.2.2 and 4.2.3, $\Phi \to \Phi'$ in the right hand side. Thanks to M. McGarrie for bringing this to my attention..
- On page 53, and in between equations 4.2.8 and 4.2.9 the cross ratio should be corrected to

  $$x = \frac{z_{12}z_{34}}{z_{13}z_{24}}$$

  Thanks to Lorenzo Battarra for bringing this to my attention.

## Section 4.3: Radial quantization

- On page 54 in the paragraph above equation (4.3.2) the sentence "We already saw that $\ell_0$ ...... cylinder." should be replaced by:

  " We already saw that $\ell_0$ was the generator of dilatations on the plane, $z \to \lambda z$ so $\ell_0 + \bar{\ell}_0$ corresponds to time translations on the cylinder."

- On page 56, the phrase "...where the last line is the desired result copied from (4.2.4)" after equation (4.3.13) should be amended to "...where the last line is the desired result copied from (4.2.4), and is valid for primary fields only."

  Thanks to B. Pioline for bringing this to my attention.

- On page 56, last line, the term "quasiprimary" should read "primary"

  Thanks to B. Pioline for bringing this to my attention.

## Section 4.6: The Hilbert space

- In equation (4.6.6) on page 60, $A\left(\frac{1}{z},\frac{1}{\bar{z}}\right) \to A\left(\frac{1}{\bar{z}},\frac{1}{z}\right)$

  Thanks to B. Pioline for bringing this to my attention.

## Section 4.7: The free boson

- Equation (4.7.9) on page 62 and the text/equations up to equation (4.7.11) should be replaced by:

$$\left\langle \prod_{i=1}^{\mathrm{N}} e^{ip_i X(z_i,\bar{z}_i)} \right\rangle = 2\pi\delta\left(\sum_{i=1}^{\mathrm{N}} p_i\right) \exp\left[-\frac{1}{2}\sum_{i,j=1}^{\mathrm{N}} p_i p_j \langle X(z_i,\bar{z}_i) X(z_j,\bar{z}_j)\rangle\right] ,$$

  where the second step in the above formula is due to the fact that we have a free (Gaussian) field theory. The momentum conserving $\delta$-function originates in the path integral from the zero mode integration of $X$. Introducing a short-distance cutoff $\epsilon$ we obtain

$$\left\langle \prod_{i=1}^{\mathrm{N}} V_{p_i}(z_i,\bar{z}_i) \right\rangle = 2\pi\delta\left(\sum_{i=1}^{\mathrm{N}} p_i\right) \exp\left[\frac{\ell_s^2}{4}\sum_{\substack{i,j=1\\ i\neq j}}^{\mathrm{N}} p_i p_j \log|z_{ij}|^2\right] \mu^{\frac{\ell_s^2}{2}\left(\sum_{i=1}^{\mathrm{N}} p_i\right)^2} \prod_{i=1}^{\mathrm{N}} \epsilon^{\frac{\ell_s^2}{2}p_i^2} ,$$

  We observe that the momentum conservation is responsible for the IR divergences to cancel from the correlator above. Moreover, the normal ordering of the exponentials removes the dependence on the short distance cutoff $\epsilon$ as this originates from self-contractions inside a single exponential. We therefore obtain

$$G_{\mathrm{N}} \equiv \left\langle \prod_{i=1}^{\mathrm{N}} V_{p_i}(z_i,\bar{z}_i) \right\rangle = 2\pi\delta\left(\sum_{i=1}^{\mathrm{N}} p_i\right) \exp\left[\frac{\ell_s^2}{4}\sum_{\substack{i,j=1\\ i\neq j}}^{\mathrm{N}} p_i p_j \log|z_{ij}|^2\right] .$$

## Section 4.8: The free fermion

- The sign of the fermion action (4.8.2) has changed to be compatible with the OPE's:

$$S = \frac{1}{2\pi}\int d^2x\ \chi^\dagger\gamma^1\partial\!\!\!/\chi = \frac{1}{2\pi}\int d^2z(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi}).$$

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.9: The conformal anomaly

- The second sentence after equation (4.9.7) should read: Substituting in (4.9.7) we find equality iff $A = -c/12$.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.12: Free fermions and O(N) affine symmetry

- The sign of (4.12.1) should be opposite:

$$S = \frac{1}{2\pi}\int d^2z\ \psi^i\bar{\partial}\psi^i\,.$$

- The previous to last sentence before equation (4.12.28) should read: By going to the basis $\psi^\pm = \psi^1 \mp i\psi^2$ it is easy to see that the $J_0^{12}$ eigenvalues of $\psi_n^\pm$ are $\pm 1$.

  Thanks to Eduard Balzin for bringing this to my attention.

### Section 4.13.1: $\mathcal{N}=(1,1)_2$ superconformal symmetry

- The sign of the fermion action should be changed in (4.13.1)

$$S=\frac{1}{2\pi\ell_s^2}\int d^2z\ \partial X\bar{\partial}X+\frac{1}{2\pi\ell_s^2}\int d^2z(\psi\bar{\partial}\psi+\bar{\psi}\partial\bar{\psi})\,.$$

- The definition of the superfield in (4.13.14) should be amended to agree with the change of sign of the action

$$\hat{X}(z,\bar{z},\theta,\bar{\theta})=X+i\theta\psi+i\bar{\theta}\bar{\psi}+\theta\bar{\theta}F\,,$$

- Equation (4.13.14) should read

$$S=\frac{1}{2\pi\ell_s^2}\int d^2z\int d\bar{\theta}d\theta\ D_\theta\hat{X}\ \bar{D}_{\bar{\theta}}\hat{X}\,.$$

Thanks to Eduard Balzin for bringing this to my attention.

### Section 4.13.3: $\mathcal{N}=(4,0)_2$ superconformal symmetry

- The OPE in equation (4.13.31) should read

$$G^\alpha(z)\bar{G}^\beta(w)=\frac{4k\delta^{\alpha\beta}}{(z-w)^3}+2\sigma^a_{\beta\alpha}\left[\frac{2J^a(w)}{(z-w)^2}+\frac{\partial J^a(w)}{(z-w)}\right]+2\delta^{\alpha\beta}\frac{T(w)}{(z-w)}+\ldots\,, \tag{4.0.1}$$

Thanks to Eduard Balzin for bringing this to my attention.

### Section 4.14: Scalars with background charge

- The sign in in front of $A$ in equation (4.14.17) must be opposite as below

$$L_m^\dagger=L_{-m}\quad,\quad J_m^\dagger=J_{-m}+A\delta_{m,0}\,.$$

Thanks to Harold Erbin for bringing this to my attention.

## Section 4.15: The CFT of ghosts

- Equations (4.15.13) and (4.15.14) on page 84, should read

$$c(z) = \sum_n z^{-n-(1-\lambda)} \, c_n \quad , \quad c_n^\dagger = c_{-n} \, ,$$

$$b(z) = \sum_n z^{-n-\lambda} \, b_n \quad , \quad b_n^\dagger = \epsilon b_{-n} \, .$$

  to account for the fact that NS and R sectors have different modding.

  Thanks to B. Pioline for bringing this to my attention.

## Section 4.16: CFT on the disk

- The sixth sentence before equation (4.16.1) should read Upon a further conformal mapping $w = \frac{i-z}{i+z}$, the upper-half plane is mapped to the unit disk (the interior of the unit circle) $|w| \leq 1$.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.18: Compact Scalars

- In equation (4.18.13) $detG \to detg$

$$\|\delta X\|^2 = \frac{1}{\ell_s^2} \int d^2\sigma \sqrt{\det g} \, (d\chi)^2 = {\sum_{m_1,m_2}}' \frac{|dA_{m_1,m_2}|^2}{\ell_s^2}$$

  Thanks to Eduard Balzin for bringing this to my attention.

- Above equation (4.18.18) the correct reference to the appendix should be :

  ...(see appendix C.4) on the integer $m$.

  Thanks to Eduard Balzin for bringing this to my attention.

- Equation (4.18.39) should read

$$Z_{\text{N,N}}(G,B) = \frac{\sqrt{\det\, G}}{\ell_s^{\text{N}}(\sqrt{\tau_2}\eta\bar{\eta})^{\text{N}}} \sum_{\vec{m},\vec{n}} e^{-\frac{\pi(G_{ij}+B_{ij})}{\tau_2 \ell_s^2}(m_i+n_i\tau)(m_j+n_j\bar{\tau})} \, . \tag{4.0.1}$$

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.20: Bosonization

- In equation (4.20.8), and (4.20,9), the $\eta$-functions are missing. The two equations should therefore read:

$$\begin{aligned} Z &= \frac{1}{2}\sum_{a,b=0}^{1}\left|\frac{\theta[^a_b]}{\eta}\right|^2 \\ &= \frac{1}{2\sqrt{2\tau_2}\eta\bar{\eta}}\sum_{a,b=0}^{1}\sum_{m,n\in\mathbb{Z}}\exp\left[-\frac{\pi}{2\tau_2}|n+\tau m|^2+i\pi(m+a)(n+b)\right] \ . \end{aligned} \quad (4.0.1)$$

$$Z_{\text{Dirac}} = \frac{1}{\sqrt{2\tau_2}\eta\bar{\eta}}\sum_{m,n\in\mathbb{Z}}\exp\left[-\frac{\pi}{2\tau_2}|n+\tau m|^2\right] \quad (4.0.2)$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.20.1: Bosonization of the bosonic ghost system

- The sign of $J^2$ in equation (4.20.11) on page 113, should be changed to minus. Also for compatibility with equation 4.14.1 we will change the sign of the background charge Q.

Therefore the rest of the section should read:

$$\hat{T} = -\frac{1}{2}:J^2:-\frac{1}{2}Q\partial J = -\frac{1}{2}(\partial\phi)^2+\frac{Q}{2}\partial^2\phi \quad , \quad Q = 1-2\lambda \ (4.20.11).$$

The boson $\phi$ has "background charge" because of the derivative term in its stress-tensor. It is described by the following action

$$S_Q = \frac{1}{2\pi}\int d^2z\left[\partial\phi\bar{\partial}\phi+\frac{Q}{2}\sqrt{g}R^{(2)}\ \phi\right] \ , (4.20.12)$$

where $R^{(2)}$ is the two-dimensional scalar curvature. Using (6.1.2 on page 154) we see that there is a background charge of $Q\chi/2$, where $\chi = 2(1-g)$ is the Euler number of the surface. This implies in practice that a correlator is non-zero if the sum of the $\phi$ charges add up to $Q\chi/2$.

A direct computation shows that $\hat{T}$ has central charge $\hat{c} = 1+3Q^2$. The original central charge of the theory was $c = \hat{c}-2$, as can be seen from (4.15.9 on page 90). Thus, we must also add an auxiliary Fermi system with $\lambda = 1$, composed of a dimension-one field $\eta(z)$ and a dimension-zero field $\xi(z)$. This system has central charge $-2$. The stress-tensor of the original system can be written as

$$T = \hat{T}+T_{\eta\xi} \ .(4.20.13)$$

Exponentials of the scalar $\phi$ have the following OPEs with the stress-tensor and the U(1) current.

$$T(z) : e^{q\phi(w)} := \left[-\frac{q(q-Q)}{2(z-w)^2} + \frac{1}{z-w}\partial_w\right] : e^{q\phi(w)} : + \ldots , \quad (4.20.14)$$

$$J(z) : e^{q\phi(w)} := \frac{q}{z-w} : e^{q\phi(w)} : \ldots \quad \to \quad [J_0, : e^{q\phi(w)} :] = q \ : e^{q\phi(w)} : . (4.20.15)$$

In terms of the new variables we can express the original $b, c$ ghosts as

$$c(z) = e^{\phi(z)}\eta(z) \quad , \quad b(z) = e^{-\phi(z)}\partial\xi(z) . (4.20.16)$$

Finally, the spin fields of $b, c$ that interpolate between NS and R sectors are given by $e^{\pm\phi/2}$ with conformal weight $-(1\mp 2Q)/8$. Note that the zero mode of the field $\xi$ does not enter the definition of $b, c$. Thus, the bosonized Hilbert space provides two copies of the original Hilbert space since any state $|\rho\rangle$ has a degenerate partner $\xi_0|\rho\rangle$.

Thanks to J. Florakis and E. Balzin for bringing this to my attention.

## Section 4.21: Orbifolds

- In the middle of equation (4.21.7) a 1/2 should be removed and the $(q\bar{q})^{\frac{1}{48}}$ should be put in the numerator.

$$\begin{aligned} Z^{\text{twisted}} &= \frac{1}{2}\text{Tr}[(1+g)q^{L_0-1/24}\ \bar{q}^{\bar{L}_0-1/24}] \\ &= (q\bar{q})^{\frac{1}{48}}\left[\prod_{n=1}^{\infty}\frac{1}{(1-q^{n-\frac{1}{2}})(1-\bar{q}^{n-\frac{1}{2}})} + \prod_{n=1}^{\infty}\frac{1}{(1+q^{n-\frac{1}{2}})(1+\bar{q}^{n-\frac{1}{2}})}\right] \\ &= \left|\frac{\eta}{\theta_4}\right| + \left|\frac{\eta}{\theta_3}\right| . \end{aligned} \quad (4.0.1)$$

Thanks to Eduard Balzin for bringing this to my attention.

- In equations (4.21.22)-(4.21.26) R should be replaced by $\frac{R}{\ell_s}$.

Thanks to Eduard Balzin for bringing this to my attention.

## Section 4.22: CFT on Other Surfaces of Euler Number Zero

- Equation (4.22.14) should read

$$Z^{\text{DN}}_{C_2,\text{boson}} = \text{N}_1\text{N}_2\ \frac{e^{-\frac{\pi t}{24}}}{\prod_{n=0}^{\infty}(1-e^{-\pi t(2n+1)})} = \text{N}_1\text{N}_2\sqrt{\frac{\eta(it)}{\vartheta_4(it)}} \quad (4.22.14)$$

Thanks to J. Florakis for bringing this to my attention.

- Equation (4.22.17) should read

$$Z^{\rm NN}_{M_2,\rm boson} = V\ CP \int_{-\infty}^{\infty} \frac{dp}{2\pi}\ \frac{e^{-2\pi\ell_s^2 p^2 t + \frac{\pi t}{12}}}{\prod_{n=1}^{\infty}(1-(-1)^n e^{-2\pi n t})} = \qquad (4.22.17)$$

$$= \frac{V\ CP}{(2\pi\ell_s)\sqrt{2t}}\frac{e^{\frac{\pi t}{12}}}{\prod_{n=1}^{\infty}(1-e^{-4\pi n t})\ (1+e^{-2\pi(2n-1)t})} = \frac{V\ CP}{(2\pi\ell_s)\sqrt{2t}\sqrt{\vartheta_3(2it)\eta(2it)}}\ .$$

Thanks to J. Florakis for bringing this to my attention.

- Equation (4.22.17) should read

$$Z^{\rm DD}_{M_2,\rm boson} = CP\delta(\Delta x)\frac{e^{\frac{\pi t}{12}}}{\prod_{n=1}^{\infty}(1+(-1)^n e^{-2\pi n t})} = CP\delta(\Delta x)\frac{\sqrt{2}\ \eta(2it)}{\sqrt{\vartheta_4(2it)\vartheta_2(2it)}} \qquad (4.22.18)$$

Thanks to J. Florakis for bringing this to my attention.

## Section: Exercises

- In exercise 4.5 the formula for the three point correlator should read

$$G^{(3)}(z_i,\bar z_i) = \frac{C_{123}}{z_{12}^{\Delta_{12}}\, z_{23}^{\Delta_{23}}\, z_{31}^{\Delta_{31}}\, \bar z_{12}^{\bar\Delta_{12}}\, \bar z_{23}^{\bar\Delta_{23}}\, \bar z_{31}^{\bar\Delta_{31}}}\ , \qquad (4.0.1)$$

Thanks to Sandy Kline for for bringing this to my attention.

# 5 Chapter 5

## Section 5.1: Physical vertex operators

- In equation (5.1.5) on page 129, $x^\nu \to X^\nu$. Therefore the equation should read

$$T(z)O(w,\bar{w}) = -ip^\mu \epsilon_{\mu\nu} \frac{\ell_s^2}{4} \frac{\bar{\partial} X^\nu V_p}{(z-w)^3} + \left(1 + \frac{\ell_s^2 p^2}{4}\right) \frac{O(w,\bar{w})}{(z-w)^2} + \frac{\partial_w O(w,\bar{w})}{z-w} + \ldots$$

- In the previous to last paragraph of page 129, the last sentence "In this context ...... by the ghosts $c(z)\bar{c}(\bar{z})$." should be replaced by :

  "In this context, the physical vertex operators are the ones we found in the old covariant case."

## Section 5.2: Calculation of tree-level tachyon amplitudes

- The footnote that is above equation (5.2.2) should read:

  This is obtained from $c(z) = \sum_{n\in Z} \frac{c_n}{z^{n-1}}$ and $\langle 0|c_{-1}c_0c_1|0\rangle = 1$, while we use $\langle 0|\bar{c}_{-1}\bar{c}_0\bar{c}_1|0\rangle = -1$

  Thanks to Eduard Balzin for bringing this to my attention.

- The first line of equation (5.2.5) on page 131 should read:

$$S_{4,\text{tree}}(s,t,u) = \frac{8\pi i}{\ell_s^2} g_s^2 (2\pi)^{26} \delta^{(26)}\left(\sum_i p_i\right) \int d^2z \; |z|^{-\ell_s^2 \frac{t}{2} - 4} \; |1-z|^{-\ell_s^2 \frac{u}{2} - 4}$$

- In equations (5.2.1), (5.2.3) and (5.2.5) $g_s$ should be replaced by $g_{\text{closed}}$ to avoid confusion. Later we use $g_s$ as the dimensionless closed string coupling, while here $g_{\text{closed}} = \frac{\kappa_{26}}{2\pi}$ where $\kappa_{26}$ is the gravitational coupling of the bosonic string. Therefore $g_{\text{closed}}$ is dimensionfull and provides the correct normalization of the vertex operators.

- Equation (5.2.10) on page 131 and the sentence that precedes it should be replaced by:

  For the ghosts inserted on the $D_2$, we have $c_n = \bar{c}_n$ and therefore

$$\langle c(z_1)c(z_2)c(z_3)\rangle_{D_2} = z_{12}z_{13}z_{23} \quad , \quad \langle c(z_1)c(z_2)\bar{c}(\bar{z}_3)\rangle_{D_2} = z_{12}(z_1-\bar{z}_3)(z_2-\bar{z}_3) \; .$$

  and so on.

- In equations (5.2.14), (5.2.15) and (5.2.16) $g_s$ should be replaced by $g_{\text{open}}$ to avoid confusion. Here $g_{\text{open}}$ is dimensionful with the same dimensions as $g_{\text{closedabove}}$

- Equation (5.2.13) on page 132 and the sentence that precedes it should be replaced by:

  Finally, for the ghosts inserted in $\mathrm{RP}_2$ we obtain

  $$\langle c(z_1)c(z_2)c(z_3)\rangle_{\mathrm{RP}_2} = z_{12}z_{13}z_{23} \quad , \quad \langle c(z_1)c(z_2)\bar{c}(\bar{z}_3)\rangle_{\mathrm{RP}_2} = z_{12}(1+z_1\bar{z}_3)(1+z_2\bar{z}_3) \ .$$

  and so on.

## Section 5.3.1: The torus

- Clarification: The last line on page 134, continuing at the top of page 135, "In our case since we have no vertex operators..... $\int d^z = \tau_2$" should be replaced by:

  In our case, since we have no vertex operators, we can directly divide by the volume of the symmetry, $\int d^z = \tau_2$, as was already specified in equations (4.17.2) and (4.17.5).

## Section 5.3.4: The Möbius strip

- Equation (5.3.24) must be slightly changed to agree with the corrected equation (3.4.8)

  $$\sum_{ij}\langle i,j|\Omega|i,j\rangle = \sum_{ij}\langle i,j|j',i'\rangle(\gamma_\Omega^{-1})_{ii'}(\gamma_\Omega)_{j'j} = \mathrm{Tr}[\gamma_\Omega^T\gamma_\Omega^{-1}] \ .$$

## Section 5.3.5: Tadpole cancelation

- Equation (5.3.29) should be replaced by the following equation and text

  $$\mathcal{T} = \mathcal{T}_{K_2} + \frac{1}{2}\mathcal{T}_{C_2} + \mathcal{T}_{M_2} = i\frac{12V_{26}}{2\pi(8\pi^2\ell_s^2)^{13}}\ \left(2^{13} - \zeta \mathrm{N}\right)^2\ \int_0^\infty d\ell \ .$$

  The factor $\frac{1}{2}$ in front of the cylinder tadpole takes into account the fact that (5.3.16) gives the tadpole of oriented open strings, and an extra orientation projection is needed to obtain the cylinder tadpole of unoriented strings.

# 6 Chapter 6

## Section 6.1: The non-linear $\sigma$-model approach.

- Clarification: In equation (6.1.12) $\nabla^2\phi$ is the same as $\Box\phi$.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 6.3: Linear dilaton and strings in $D < 26$ dimensions

- The sentence above equation (6.3.3) should be modified as follows:

  "Using also the $L_0$ eigenvalues in (4.14.10 on page 88), we find that the operators that satisfy the physical state condition $L_0 = 1$ are...."

  Also, below equation (6.3.3) the two sentences "Such a background would have been a tachyon ....... and it becomes massless in $D = 1$." should be deleted.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 6.4: T-duality in non-trivial backgrounds

- Equation (6.4.2) should read

$$\tilde{G}_{00} = \frac{1}{G_{00}} \quad , \quad \tilde{G}_{0i} = \frac{B_{0i}}{G_{00}} \quad , \quad \tilde{B}_{0i} = \frac{G_{0i}}{G_{00}} \quad , \quad \tilde{\Phi} = \Phi - \frac{1}{2}\log G_{00} \; ,$$

  Thanks to Eduard Balzin who brought this to my attention.

# 7 Chapter 7

## Section 7.1: $\mathcal{N} = (1,1)_2$ superconformal symmetry

- The spacetime indices in several equations must be placed correctly for summation

$$S_P^{II} = \frac{1}{4\pi\ell_s^2}\int d^2\sigma\sqrt{g}\,\Big[g^{ab}\partial_a X_\mu \partial_b X^\mu + i\bar{\psi}_\mu \partial\!\!\!/ \psi^\mu + i(\bar{\chi}_a\gamma^b\gamma^a\psi_\mu)\left(\partial_b X^\mu - \frac{i}{4}\bar{\chi}_b\psi^\mu\right)\Big]\,. \tag{7.0.1}$$

$$G_{\text{matter}} = i\frac{\sqrt{2}}{\ell_s^2}\psi_\mu\partial X^\mu \quad , \quad \bar{G}_{\text{matter}} = i\frac{\sqrt{2}}{\ell_s^2}\bar{\psi}_\mu\bar{\partial}X^\mu\,. \tag{7.0.5}$$

$$G_{\text{matter}} = i\frac{\sqrt{2}}{\ell_s^2}\psi_\mu\partial X^\mu \quad , \quad T_{\text{matter}} = -\frac{1}{\ell_s^2}\partial X_\mu\partial X^\mu - \frac{1}{2\ell_s^2}\psi_\mu\partial\psi^\mu\,, \tag{7.0.7}$$

- The ghost supercurrent in equation (7.1.8) on page 156 should read:

$$G_{\text{ghost}} = -c\,\partial\beta - 2\gamma\,b - \frac{3}{2}\partial c\,\beta$$

Thanks to B. Pioline for bringing this to my attention.

## Section 7.2: Closed (type II) superstrings

- Footnote 1 should read

  There is a subtlety here concerning the super-light-cone gauge. If $\psi^+$ for example has NS boundary conditions, then it can be set to zero. If it has R boundary conditions, then it can be set to zero except for its zero mode, $\psi^+ = \psi_0^+ = \frac{\Gamma^+}{\sqrt{2}}$. A similar remark applies to $\bar{\psi}^+$.

## Section 7.3: Type-I superstrings

- The text from above equation (7.3.3) until after (7.3.4) should be modified as follows:

  Similarly, in the R$\bar{\text{R}}$ sector, supersymmetry will imply that $\epsilon_{\text{R}} = 1$. You are asked in exercise 7.8 on page 184 to show, using (7.3.2), that the IIB bispinor (7.2.17) transforms under $\Omega$ as

  $$\Omega\ F_{\alpha\beta}\ \Omega^{-1} = \epsilon_{\text{R}}\Omega S_\alpha \Omega^{-1}\Gamma^0_{\beta\gamma}\Omega\tilde{S}_\gamma\Omega^{-1} = \epsilon_{\text{R}}\tilde{S}_\alpha\Gamma^0_{\beta\gamma}S_\gamma = -\epsilon_{\text{R}}\Gamma^0_{\beta\gamma}S_\gamma\tilde{S}_\alpha =$$

  $$= -\epsilon_{\text{R}}\Gamma^0_{\beta\gamma}F_{\gamma\delta}\Gamma^0_{\delta\alpha} = -\epsilon_{\text{R}}\left(\Gamma^0\right)^T_{\alpha\delta}F^T_{\delta\gamma}\left(\Gamma^0\right)^T_{\gamma\beta} = -\epsilon_{\text{R}}\ (\Gamma^0\ F^T\ \Gamma^0)_{\alpha\beta}\ \ .$$

  We have added the extra sign $\epsilon_R$ above that comes from the transformation of the ground state. We have also treated the left and right spinors $S_\alpha$ and $\tilde{S}_\alpha$ as anticommuting with each other.

  Using the $\Gamma$-matrix properties $(\Gamma^\mu)^T = \Gamma^0\Gamma^\mu\Gamma^0$ and $(\Gamma^0)^2 = -1$, we find that the IIB forms transform as

  $$F_{\mu_1\mu_2\cdots\mu_k} \to \epsilon_{\text{R}}\ (-1)^k F_{\mu_k\mu_{k-1}\cdots\mu_1} = \epsilon_{\text{R}}\ (-1)^{\frac{k(k+1)}{2}} F_{\mu_1\mu_2\cdots\mu_k}\quad ,\quad k\ \text{ odd} \qquad (7.3.4)$$

  Therefore, for $\epsilon_{\text{R}} = 1$, the two-index antisymmetric tensor survives, but the scalar and the four-index self-dual antisymmetric tensor are projected out.

  Thanks to J. Florakis for bringing this to my attention.

- Equation (7.3.14) must be changed to

  $$[\lambda_1, \lambda_2] = -\epsilon^2_{\text{NS}}\ \gamma_\Omega\ [\lambda_1^T, \lambda_2^T]\ (\gamma_\Omega)^{-1} = -\ \gamma_\Omega\ [\lambda_1, \lambda_2]^T\ (\gamma_\Omega)^{-1}\ .$$

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 7.4: Heterotic superstrings

- The third paragraph of this section should read:

  To remove the tachyon, we will also impose the usual GSO projection on the left, namely $(-1)^{F_L} = -1$. Here, we will have two sectors, generated by the left-moving fermions, the NS sector (space-time bosons) and the R sector (space-time fermions). Also the non-compact space-time dimension is ten, the $\phi^I$ being compact ("internal") coordinates.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 7.5: Superstring vertex operators

The whole sections is reworked below, in order to correct inconsistencies with other parts of the book, and to also introduce normalized vertex operators for superstring theories. Thanks to B. Piolin and E. Balzin for pointing out errors.

In analogy with the bosonic string, the vertex operators must be primary states of the superconformal algebra. We first describe the left-moving part of the superstring and therefore use chiral superfield language (see (4.13.13),(4.13.14 on page 79) where[1]

$$\hat{X}^\mu(z,\theta) = X^\mu(z) + \frac{i}{\sqrt{2}}\theta\psi^\mu(z)\,. \tag{7.0.1}$$

The left-moving vertex operators can be written in the form:

$$\int dz \int d\theta\; V(z,\theta) = \int dz \int d\theta \left(\frac{i}{\sqrt{2}}V_{-1}(z) + \theta V_0(z)\right) = \int dz\; V_0\,. \tag{7.0.2}$$

The conformal weight of $V_{-1}$ is $\frac{1}{2}$ while that of $V_0$ is 1. The integral of $V_0$ has conformal weight zero. For the massless space-time bosons the vertex operator is[2]

$$V^{\rm boson}(\epsilon,p,z,\theta) = \epsilon_\mu : D\hat{X}^\mu\; e^{ip\cdot\hat{X}} :, \tag{7.0.3}$$

$$V^{\rm boson}_{-1}(\epsilon,p,z) = \epsilon_\mu\psi^\mu e^{ip\cdot X} \quad , \quad V^{\rm boson}_0(\epsilon,p,z) = \epsilon_\mu : \left(\partial X^\mu - \frac{i}{2}p\cdot\psi\;\psi^\mu\right) e^{ip\cdot X} :, \tag{7.0.4}$$

where $\epsilon\cdot p = 0$.

We would like to present the vertex operators in the covariant quantization. The reason is that the fermion vertex has a simple form only in the modern covariant quantization. It is useful to use the bosonization of the $\beta-\gamma$ system described in section (4.20.1 on page 106) in terms of a scalar field $\phi$ with background charge $Q=2$, and the $\eta-\xi$ system as

$$\gamma(z) = e^{\phi(z)}\eta(z) \quad , \quad \beta(z) = e^{-\phi(z)}\partial\xi(z)\,. \tag{7.0.5}$$

$\eta$ has dimension one while $\xi$ has dimension zero. In general, the vertex operator $: e^{q\phi} :$ has conformal weight $-q-\frac{q^2}{2}$. The spin fields of $\beta,\gamma$ that interpolate between NS and R sectors are given by $e^{\pm\phi/2}$ with conformal weight $-5/8$ and $3/8$. It should be noted that the zero mode of the $\xi$ field does not appear in the bosonization of $\beta-\gamma$ system. It introduces an extra redundancy in the new Hilbert space.

There is a subtlety in the case of fermionic strings having to do with the $\beta,\gamma$ system. As we have seen, in the bosonized form, the presence of the background charge alters the charge neutrality condition[3]. This is related to the existence of super-moduli and

[1]Note that we use a normalization of fermions matching that of bosons, as explained in section 4.13.1

[2]See also exercise (4.39) on page 122.

[3]The notion of the charge neutrality condition was described in section 4.14 on page 82. Its interpretation in terms of zero modes and the index can be found in section 4.15 on page 84.

super-killing spinors. Therefore, depending on the correlation function and surface we must have different representatives for the vertex operators of a given physical state with different $\phi$-charges. The precise condition, derived in section 4.20.1 is that the sum of $\phi$ charges must be equal to the Euler number of the surface, $\chi$.

Different representatives are constructed as follows. Consider a physical vertex operator with $\phi$ charge $q$, $V_q$. It is BRST invariant, $[Q_{\rm BRST}, V_q] = 0$. We can construct another physical vertex operator representing the same physical state but with charge $q+1$. It is $V_{q+1} = [Q_{\rm BRST}, \xi V_q]$ and has charge $q+1$ since $Q_{\rm BRST}$ carries charge 1. Since it is a BRST commutator, $V_{q+1}$ is also BRST-invariant. However, we have seen that states that are BRST commutators of physical states are spurious. This is not the case here since the $\xi$ field appears in the commutator and its zero mode lies outside the ghost Hilbert space. The different $\phi$ charges are usually called "pictures" in the literature.

In the covariant setup, the massless NS vertex operator becomes [4]

$$V_{-1}^{\rm boson}(\epsilon, p, z) = e^{-\phi(z)} \epsilon \cdot \psi \; e^{ip\cdot X} \; . \tag{7.0.6}$$

We can construct another equivalent vertex operator in the zero-picture as

$$V_0^{\rm boson}(\epsilon, p, z) = -\frac{i}{\sqrt{2}}[Q_{\rm BRST}, \xi(z) e^{-\phi(z)} \epsilon \cdot \psi \; e^{ip\cdot X}] = \epsilon_\mu \left( \partial X^\mu - \frac{i}{2} p \cdot \psi \psi^\mu \right) \; e^{ip\cdot X} \; . \tag{7.0.7}$$

The space-time fermion vertex operators can only be constructed in the covariant formalism. For the massless states ($p^2 = 0$), in the canonical $-\frac{1}{2}$ picture they are of the form

$$V_{-1/2}^{\rm fermion}(u, p, z) = u^\alpha(p) : e^{-\phi(z)/2} \; S_\alpha(z) \; e^{ip\cdot X} :, \tag{7.0.8}$$

$S_\alpha$ is the spin field of the fermions $\psi^\mu$ forming an $\mathrm{O}(10)_1$ current algebra, with weight 5/8 (see section 4.12 on page 71). The total conformal weight of $V_{-1/2}$ is 1. Finally, $u^\alpha$ is a spinor wave-function, satisfying the massless Dirac equation $\not{p} u = 0$.

The $\frac{1}{2}$ picture for the fermion vertex can be computed to be

$$\begin{aligned} V_{1/2}^{\rm fermion}(u, p) &= [Q_{\rm BRST}, \xi(z) V_{-1/2}^{\rm fermion}(u, p, z)] \\ &= u^\alpha(p) \; e^{\phi/2} \; (\Gamma^\mu)_{\alpha\beta} \; S^\beta \; \partial X^\mu \; e^{ip\cdot X} + \cdots \; , \end{aligned} \tag{7.0.9}$$

where the ellipsis involves terms that do not contribute to four-point amplitudes. The ten-dimensional space-time supersymmetry charges can be constructed from the fermion vertex at zero momentum,

$$Q_\alpha = \frac{1}{2\pi i} \oint dz \;\; : e^{-\phi(z)/2} \; S_\alpha(z) : \; . \tag{7.0.10}$$

It transforms fermions into bosons and vice versa

$$[Q_\alpha, V_{-1/2}^{\rm fermion}(u, p, z)] = V_{-1}^{\rm boson}(\epsilon^\mu = u^\beta \gamma^\mu_{\beta\alpha}, p, z) \, , \tag{7.0.11}$$

$$[Q_\alpha, V_0^{\rm boson}(\epsilon, p, z)] = V_{-1/2}^{\rm fermion}(u^\beta = ip^\mu \epsilon^\nu (\gamma_{\mu\nu})^\beta_\alpha, p, z) \, . \tag{7.0.12}$$

There are various pictures for the supersymmetry charges also.

[4] We are using the same symbol for the vertex operator both in the old covariant and the BRST formulation.

### Section 7.5.1: Open superstring vertex operators

In the open string, the vertex operators described above are the whole story with the caveat that we must use a different normalization for the momentum in order to accommodate the different size of the string. The gauge boson vertex operators are

$$V_{-1}^{a,\mu} = g_{\rm open}\ \lambda^a\ \psi^\mu\ e^{ip\cdot X} \quad , \quad V_0^{a,\mu} = \frac{ig_{\rm open}}{\sqrt{2}\ell_s}\ \lambda^a\ [\partial_\tau X^\mu - 2ip\cdot\psi\psi^\mu]\ e^{ip\cdot X} \tag{7.0.13}$$

where we have included the proper normalizations necessary for unitarity.

The gaugino vertex operators are

$$V_{-1/2}^{\alpha,a} = g_{\rm open}\sqrt{\ell_s}\ \lambda^a\ \ :e^{-\phi/2}\ S_\alpha\ e^{ip\cdot X}: \tag{7.0.14}$$

$g_{\rm open}$ is dimensionfull and is given by

$$g_{\rm open} = 2^{\frac{3}{4}}(2\pi)^{\frac{7}{2}}\ell_s^4\ g_s \tag{7.0.15}$$

### Section 7.5.2: Type II superstring vertex operators

In the type II closed strings the vertex operators are products of two copies of the supersymmetric chiral vertex operators. For NS-NS massless states they are

$$V_{-1,-1}^{\mu\nu} = g_{\rm closed}\ \psi^\mu\bar{\psi}^\nu\ e^{ip\cdot X} \quad , \quad V_{0,0}^{\mu\nu} = \frac{2g_{\rm closed}}{\ell_s^2}\left[\partial_z X^\mu - \frac{i}{2}p\cdot\psi\psi^\mu\right]\left[\partial_{\bar{z}} X^\mu - \frac{i}{2}p\cdot\bar{\psi}\bar{\psi}^\nu\right]\ e^{ip\cdot X} \tag{7.0.16}$$

while for the RR massless bosons we obtain

$$V_{-1/2,-1/2}^{\alpha,\beta} = \frac{g_{\rm closed}\ell_s}{2}\ \ :e^{-\phi/2-\bar{\phi}/2}\ S_\alpha\bar{S}_\beta\ e^{ip\cdot X}: \tag{7.0.17}$$

Finally for the NS-R gravitino we have

$$V_{-1,-1/2}^{\mu,\alpha} = \frac{g_{\rm closed}\sqrt{\ell_s}}{\sqrt{2}} :\psi^\mu e^{-\bar{\phi}/2}\ \bar{S}_\alpha\ e^{ip\cdot X}: \tag{7.0.18}$$

$$V_{0,-1/2}^{\mu,\alpha} = \frac{g_{\rm closed}}{\sqrt{\ell_s}} : \left[\partial_z X^\mu - \frac{i}{2}p\cdot\psi\psi^\mu\right] e^{-\bar{\phi}/2}\ \bar{S}_\alpha\ e^{ip\cdot X}: \tag{7.0.19}$$

$g_{\rm closed}$ is dimensionfull and is defined in terms of the ten-dimensional gravitational coupling as

$$g_{\rm closed} = \frac{\kappa}{2\pi} = 4\pi^{\frac{5}{2}}\ell_s^4 g_s \tag{7.0.20}$$

It is the same for all closed superstring theories.

### Section 7.5.3: Heterotic string vertex operators

For the heterotic string theory, the left-moving part is supersymmetric and the vertex operators are as discussed earlier. The right-moving part is non-supersymmetric and the vertex operators are those discussed in chapter 5 on page 126. In particular, for the massless states they are currents multiplied by exponentials. We have only left-moving pictures here.

The massless vertex operators for the graviton, antisymmetric tensor and dilaton are

$$V^{\mu\nu}_{-1} = \frac{\sqrt{2}g_{\rm closed}}{\ell_s}\psi^\mu\bar\partial X^\nu e^{ip\cdot X} \quad , \quad V^{\mu\nu}_{0} = \frac{2g_{\rm closed}}{\ell_s^2}(\partial X^\mu - \frac{i}{2}p\cdot\psi\psi^\mu)\bar\partial X^\nu e^{ip\cdot X} \ . \tag{7.0.21}$$

while for the gravitino

$$V^{\mu,\alpha}_{-1/2} = \frac{g_{\rm closed}}{\sqrt{\ell_s}} : e^{-\phi/2}\ S_\alpha(z)\bar\partial X^\mu\ e^{ip\cdot X}\ . \tag{7.0.22}$$

For the gauge bosons we obtain

$$V^{a,\mu}_{-1} = \frac{\sqrt{2}g_{\rm closed}}{\ell_s}\psi^\mu\bar J^a e^{ip\cdot X} \quad , \quad V^{a,\mu}_{0} = \frac{2g_{\rm closed}}{\ell_s^2}(\partial X^\mu - \frac{i}{2}p\cdot\psi\psi^\mu)\bar J^a e^{ip\cdot X} \ . \tag{7.0.23}$$

where $\bar J^a$ is an anti-holomorphic $O(32)$ or $E_8\times E_8$ current

Finally for the gaugini

$$V^{a,\alpha}_{-1/2} = \frac{g_{\rm closed}}{\sqrt{\ell_s}} : e^{-\phi/2}\ S_\alpha(z)\bar J^a\ e^{ip\cdot X} :\ . \tag{7.0.24}$$

In the heterotic case, $g_{\rm closed}$ is still given by (7.0.20).

## Section 7.6.3: The type-I superstring

- Clarification of the starting point of equation (7.6.11) and associated corrections to the same equations:

$$\Lambda^{\rm NS}_{M_2} = -i\zeta {\rm N} V_{10}\int_0^\infty \frac{dt}{8t}\,\frac{\sum_{b=0}^1\ (-1)^b\ \hat\vartheta^4[^0_b]\,(it)}{(8\pi^2\ell_s^2 t)^5\ \hat\eta^{12}\,(it)} =$$

According to the discussion above it, the more obvious starting point should be

$$\Lambda^{\rm NS}_{M_2} = -i\zeta\epsilon_{NS}{\rm N}V_{10}\int_0^\infty \frac{dt}{8t}\,\frac{1}{(8\pi^2\ell_s^2t)^5}\ Z_B\ Z_F \tag{7.0.1}$$

where the fermionic contribution according to (7.3.5)-(7.3.7) are

$$Z_F = \sum_{b=0}^1\ (-1)^b\ \frac{\vartheta^4[^0_b]\,(-q)}{\hat\eta^4} = \frac{\vartheta_2^4(-q)}{\hat\eta^4} = 2^4(-q)^{\frac12}q^{-\frac16}\prod_{n=1}^\infty(1+(-q)^n)^8$$

The factor of $(-q)^{\frac12}$ comes from the lowest level of the vector representation that survives the GSO Projection (and therefore transforms under $\Omega$) while the $q^{-\frac16}$

comes from the $\hat{\eta}^4$ function. According to (7.3.7) $(-q)^{\frac{1}{2}} = i\ q^{\frac{1}{2}}$ and splitting the product into odd and even integers we obtain

$$Z_F = i\ 2^4 (q)^{\frac{1}{2}-\frac{1}{6}} \prod_{n=1}^{\infty} \left(1+q^{2n}\right)^8 \left(1-q^{2n-1}\right)^8$$

Using

$$\prod_{n=1}^{\infty} \left(1+q^{2n}\right)^8 = 2^{-4} q^{-\frac{2}{3}} \frac{\vartheta_2^4(q^2)}{\eta^4(q^2)} \quad , \quad \prod_{n=1}^{\infty} \left(1-q^{2n-1}\right)^8 = q^{\frac{1}{3}} \frac{\vartheta_4^4(q^2)}{\eta^4(q^2)}$$

we finally obtain for the fermionic contribution

$$Z_F = i\ \frac{\vartheta_2^4(q^2)}{\eta^4(q^2)} \frac{\vartheta_4^4(q^2)}{\eta^4(q^2)}$$

The bosonic contribution according to (7.3.5)-(7.3.7) is

$$Z_B = \frac{1}{\hat{\eta}^8\,(it)} = \frac{1}{q^{\frac{1}{3}} \prod_{n=1}^{\infty} (1-(-q)^n)^8} = \frac{1}{q^{\frac{1}{3}} \prod_{n=1}^{\infty} (1-q^{2n})^8 (1+q^{2n-1})^8}$$

where we have split the product into odd and even integers. Using

$$\prod_{n=1}^{\infty} \left(1+q^{2n-1}\right)^8 = q^{\frac{1}{3}} \frac{\vartheta_3^4(q^2)}{\eta^4(q^2)} \quad , \quad \prod_{n=1}^{\infty} \left(1-q^{2n}\right)^8 = q^{-\frac{2}{3}} \eta^8(q^2)$$

We therefore obtain

$$Z_B = \frac{1}{\theta_3^4(q^2)\eta^4(q^2)}$$

in agreement with (C.28) of appendix C. Putting everything together we obtain

$$Z_B Z_F = i\ \frac{\vartheta_2^4(q^2)\ \vartheta_4^4(q^2)}{\theta_3^4(q^2)\ \eta^{12}(q^2)} \equiv i\ \frac{\vartheta_2^4(2it)\ \vartheta_4^4(2it)}{\theta_3^4(2it)\ \eta^{12}(2it)}$$

We now substitute in (7.0.1) above to obtain

$$\Lambda_{M_2}^{\mathrm{NS}} = -i\zeta(i\epsilon_{NS}) \mathrm{N} V_{10} \int_0^{\infty} \frac{dt}{8t} \frac{1}{(8\pi^2 \ell_s^2 t)^5} \frac{\vartheta_2^4(2it)\ \vartheta_4^4(2it)}{\theta_3^4(2it)\ \eta^{12}(2it)} \tag{7.0.2}$$

Since $\epsilon_{NS} = -i$ from equation (7.3.15) in the book we finally obtain

$$\Lambda_{M_2}^{\mathrm{NS}} = -i\zeta \mathrm{N} V_{10} \int_0^{\infty} \frac{dt}{8t} \frac{1}{(8\pi^2 \ell_s^2 t)^5} \frac{\vartheta_2^4(2it)\ \vartheta_4^4(2it)}{\theta_3^4(2it)\ \eta^{12}(2it)}$$

that should replace the second equation of (7.6.11) in the book.

We now proceed to the transverse channel by doing the modular transformation

$$\vartheta_3(2it) = \frac{\vartheta_3\left(\frac{i}{2t}\right)}{\sqrt{2t}} \quad , \quad \vartheta_2(2it) = \frac{\vartheta_4\left(\frac{i}{2t}\right)}{\sqrt{2t}} \quad , \quad \vartheta_4(2it) = \frac{\vartheta_2\left(\frac{i}{2t}\right)}{\sqrt{2t}} \quad , \quad \eta(2it) = \frac{\eta\left(\frac{i}{2t}\right)}{\sqrt{2t}}$$

and changing variable to $\ell = \frac{\pi}{4t}$ to finally obtain

$$\Lambda^{\rm NS}_{M_2} = -i\zeta{\rm N}\frac{2^6 V_{10}}{(8\pi^2\ell_s^2)^5}\int_0^\infty d\ell \ \frac{\vartheta_2^4\left(\frac{2i\ell}{\pi}\right)\ \vartheta_4^4\left(\frac{2i\ell}{\pi}\right)}{\theta_3^4\left(\frac{2i\ell}{\pi}\right)\ \eta^{12}\left(\frac{2i\ell}{\pi}\right)}$$

which replaces the third equation (7.6.11).

Along similar lines equation (7.6.12) should read

$$\begin{aligned}\Lambda^{\rm R}_{M_2} &= -i\epsilon_{\rm R}\zeta{\rm N}V_{10}\int_0^\infty \frac{dt}{8t}\ \frac{\sum_{b=0}^1\ (-1)^b\ \hat\vartheta^4[^1_b]\,(it)}{(8\pi^2\ell_s^2 t)^5\ \hat\eta^{12}\,(it)} = \\ &= -i\epsilon_{\rm R}\zeta{\rm N}V_{10}\int_0^\infty \frac{dt}{8t}\ \frac{\vartheta_2(2it)^4\vartheta_4(2it)^4}{(8\pi^2\ell_s^2 t)^5\ \eta(2it)^{12}\vartheta_3(2it)^4} \\ &= -i\epsilon_{\rm R}\zeta{\rm N}\frac{2^6 V_{10}}{8\pi(8\pi^2\ell_s^2)^5}\int_0^\infty d\ell\ \frac{\vartheta_2\left(2i\frac{\ell}{\pi}\right)^4\vartheta_4\left(2i\frac{\ell}{\pi}\right)^4}{\eta\left(2i\frac{\ell}{\pi}\right)^{12}\vartheta_3\left(2i\frac{\ell}{\pi}\right)^4}\ .\end{aligned}$$

Thanks to Eduard Balzin for asking the relevant question.

## Section 7.9: Anomalies

- The definition of the gauge transformation in the sentence above equation (7.9.2) should be modified to:

  $\delta_\Lambda A = D\Lambda = d\Lambda + [A,\Lambda]$

  Equation (7.9.2) should be modified to

  $$\delta_\Lambda\Gamma^{\rm eff} = {\rm Tr}\int\ D_\mu\Lambda\ \frac{\delta\Gamma^{\rm eff}}{\delta A_\mu} = -{\rm Tr}\int\ \Lambda\ D_\mu\frac{\delta\Gamma^{\rm eff}}{\delta A_\mu} = -\int\ {\rm Tr}\ [\Lambda\ D_\mu J^\mu]\,,$$

  and equation (7.9.3) should be modified to

  $$\delta_{\rm diff}\Gamma^{\rm eff} = \int\ (\nabla^\mu\epsilon^\nu + \nabla^\nu\epsilon^\mu)\frac{\delta\Gamma^{\rm eff}}{\delta g_{\mu\nu}} = -2\int\epsilon^\mu\nabla_\nu T^{\mu\nu}\,.$$

  Thanks to Eduard Balzin for bringing this to my attention.

- Equation (7.9.6) should read

  $$\begin{aligned}\delta\Gamma|_{\rm mixed}\ \sim\ &\int d^{10}x\ \left[e_1{\rm Tr}[\Lambda F_0]{\rm Tr}[R_0^4] + e_2{\rm Tr}[\Theta R_0]{\rm Tr}[F_0^4] +\right.\\ &\left.+e_3{\rm Tr}[\Theta R_0]({\rm Tr}[F_0^2])^2 + e_4{\rm Tr}[\Lambda F_0]({\rm Tr}[R_0^2])^2\right]\ . \qquad (7.0.1)\end{aligned}$$

  Thanks to Eduard Balzin for bringing this to my attention.

# 8 Chapter 8

## Section 8.2: Open strings and T duality

- Equation (8.2.10) should read

$$\tilde{X}^9(\tau,\sigma) = 2\ell_s^2 p^9 \sigma - \sqrt{2}\ell_s \sum_{k\in\mathbb{Z}} \frac{\alpha^9{}_k}{k} e^{-ik\tau} \sin(k\sigma)\ .$$

- Equation (8.2.12) should read

$$\tilde{X}^9(\sigma=\pi) - \tilde{X}^9(\sigma=0) = \tilde{R}\ (2\pi n + \chi_j - \chi_i) \sim \tilde{R}\ (-\chi_i + \chi_j)\ .$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 8.4: D-branes and RR charges

- Equation (8.4.9) should read

$$S_p = T_p \int d^{p+1}\xi\ e^{-\Phi}\sqrt{\det \hat{G}} + iT_p \int\ C_{p+1} \quad ,$$

Thanks to Eduard Balzin for bringing this to my attention.

- Equation (8.4.15) should read

$$R_p' = \frac{\ell_s^2}{R_p} \quad , \quad e^{-2\Phi'}R_p' = e^{-2\Phi}R_p \to g_s' = g_s\frac{\ell_s}{R_p} \quad , \quad T_p' = T_p\frac{R_p}{\ell_s}\ ,$$

Thanks to Eric Perlmutter for bringing this to my attention.

## Section 8.5.1: The Dirac-Born-Infeld action

- Equations (8.5.1)-(8.5.4) should read

$$S_{BI} = T_9 \int d^{10}x\ e^{-\Phi} \sqrt{-\det(\eta_{\mu\nu} + 2\pi\ell_s^2 F_{\mu\nu})} \quad (8.5.1)$$

$$S_{D_0} = T_0 \int d\tau\ e^{-\Phi} \sqrt{-G_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \quad (8.5.2)$$

$$S_p = T_p \int d^{p+1}\xi\ e^{-\Phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + 2\pi\ell_s^2\ F_{\alpha\beta})} + T_p \int\ C_{p+1} \quad (8.5.3)$$

$$S_B = \frac{1}{4\pi\ell_s^2} \int_{M_2} d^2\xi\ \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu + \int_{B_1} ds\ A_\mu \partial_s x^\mu \quad (8.5.4)$$

Thanks to Eduard Balzin for bringing this to my attention.

- Equation (8.5.6) should have the opposite sign and be without the $i$:

$$\delta S_B = \frac{1}{2\pi\ell_s^2} \int_{B_1} ds\ \Lambda_\mu \partial_s x^\mu \quad .$$

Thanks to Eduard Balzin for bringing this to my attention.

- Equation (8.5.9) should read

$$S_p = T_p \int d^{p+1}\xi\ e^{-\Phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + \mathcal{F}_{\alpha\beta})} + T_p \int\ C_{p+1} \quad .$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 8.5.2: Anomaly related terms

- Equations (8.5.10) and (8.5.12) should read

$$S_p = T_p \int d^{p+1}\xi \; e^{-\Phi}\sqrt{-\det(\hat{G}+\mathcal{F})} + T_p \int \; C \wedge \mathrm{Tr}[e^{\mathcal{F}}] \wedge \mathcal{G} \;, \quad (8.5.10)$$

$$S_1 = \frac{1}{2\pi\ell_s^2}\left[\int d^2\xi \; e^{-\Phi} \; \sqrt{-\det(\hat{G}+\mathcal{F})} + \int (C_2 + C_0\mathcal{F})\right] \quad (8.5.12)$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 8.7: T-duality and orientifolds.

- The text starting at the paragraph "We will reinterpret..." till after equation (8.7.10) was changed as follows:

We will reinterpret the tadpole cancellation of the type I string in ten dimensions. The Klein bottle introduces tadpoles that are due to one non-dynamical plane. This is the $O_9$ orientifold plane. To calculate its tension we reason as follows. The 32 $_D9$ branes that cancel tadpoles generate an SO(32) group because of the $\Omega$ projection. They are equivalent to 16 unitary branes and their images under $\Omega$. This tension (and the associated charge) is cancelled by the $O_9$ plane. Therefore the $O_9$ plane has a tension

$$T_9^{\mathrm{O}} = -16 \; T_9 \quad (8.0.1)$$

Its energy and charge is cancelled by the $D_9$ branes. The vacuum therefore contains 16 $D_9$ branes and their images under $\Omega$ that cancel the orientifold charge and maintain a flat space.

Unlike D-branes, O-planes are non-dynamical in the sense that they cannot fluctuate and therefore cannot carry degrees of freedom. This is as well, since a fluctuating negative tension object has necessarily negative-norm states.

One-loop amplitudes were already calculated in section 5.3 on page 133 for the bosonic string and in section 7.6 on page 170 for the superstring. In particular, in the superstring case we have found that the tadpoles cancel in the presence for N=$2^5$ $D_9$ branes and their $\Omega$-images. T-dualizing on a circle, we will need $2^4$ $D_8$ branes to cancel them since the other $2^4$ are images.

On the other hand we know have two $O_8$ planes as the orientifold projection now includes a space inversion in the ninth direction. Therefore,

$$T_8^{\mathrm{O}} = -8 \; T_8 \quad (8.0.2)$$

Continuing further the T-dualization we conclude that the tension of an $\mathrm{O}_p$ plane is

$$T_p^{\mathrm{O}} = -2^{p-5}\, T_p \tag{8.0.3}$$

The effective action of an $\mathrm{O}_p$ plane is then

$$S_{O_p} = -2^{p-5} T_p \left[ \int d^{p+1}\xi\; e^{-\Phi} \sqrt{-\det g} + \int d^{p+1}\xi\; C_{p+1} \right] . \tag{8.0.4}$$

We should again stress that orientifold planes have no (intrinsic) dynamical degrees of freedom. They just carry charges (energy, R-R charge, etc.)

## Section 8.8.1: The supergravity solutions

- Equation (8.8.9) should read

$$F_{r01\cdots p} = \sqrt{1 + \frac{r_0^{7-p}}{L^{7-p}}}\, \frac{H_p'(r)}{H_p^2(r)} \,.$$

Thanks to Eduard Balzin for bringing this to my attention.

## Section 8.8.2: Horizons and singularities

- The text after equation (8.8.18) should read:

There is an inner Killing horizon at $\rho = r_-$ and an outer horizon at $\rho = r_+$. There is no curvature singularity at the outer horizon. There is however a curvature singularity at the inner horizon. We may calculate the scalar curvature in the string metric for an extremal brane ($r_0 = 0$) to be

$$R_\sigma = \frac{(p+1)(3-p)(p-7)^2}{4\; r^{\frac{p-3}{2}}\; L^{\frac{7-p}{2}}} \left[1 + \mathcal{O}(r^{7-p})\right] \simeq \frac{(p+1)(3-p)(p-7)^2}{4\; \left(\rho^{7-p} - r_-^{7-p}\right)^{\frac{p-3}{2(7-p)}}\; L^{\frac{7-p}{2}}} + \cdots \,.$$

The exact form of the curvature scalar for a non-extremal D-brane is

$$R_\sigma = -\frac{(p-3)(p-7)^2}{4} \frac{L^{7-p}}{r^{23-3p} H_p(r)^{\frac{5}{2}}} \left(4(rr_0)^{7-p} + L^{7-p}((p+1)r^{7-p} - (p-3)r_0^{7-p})\right)$$

with

$$H_p(r) = 1 + \frac{L^{7-p}}{r^{7-p}}$$

Thanks to Kostas Anagnostopoulos for bringing this to my attention.

- The text below equation (8.8.19) should read

  It therefore seems that there is no singularity for $p = 3$. For $7 > p > 3$ there is a curvature singularity at $\rho = r_-$. On the other hand, for $-1 \leq p < 3$ the curvature scalar is regular at $\rho = r_-$.

  Thanks to Eduard Balzin for bringing this to my attention.

## Section 8.8.3: The extremal branes and their near horizon geometry

- In equation (8.8.32) there should be no semicolon in the Riemann tensor:

$$\ell_s^4 \; R_{\mu\nu\;\rho\sigma} R^{\mu\nu\;\rho\sigma} = \begin{cases} c_+(p) \left(\frac{r}{\ell_s}\right)^{-4} \left(\frac{L}{r}\right)^{2(7-p)} + \cdots, & r \gg L, \\ c_-(p) \left(\frac{r}{\ell_s}\right)^{-4} \left(\frac{r}{L}\right)^{7-p} + \cdots, & r \ll L. \end{cases}$$

  Thanks to Kostas Anagnostopoulos for bringing this to my attention.

- In equation (8.8.34) there should be no semicolon in the Riemann tensor:

$$\ell_s^4 \; R_{\mu\nu\;\rho\sigma} R^{\mu\nu\;\rho\sigma} \simeq 80 \left(\frac{\ell_s}{L}\right)^4 \sim \frac{1}{\lambda}\,.$$

  Thanks to Kostas Anagnostopoulos for bringing this to my attention.

## Exercises (Chapter 8)

- Equation (8.1E) should read

$$S = \frac{1}{g_{\mathrm{s}}\ \ell_s} \int dt\ \mathrm{Tr}\left[ (\dot{X}^I + [A_t, X^I])^2 + \frac{1}{2(2\pi\ell_s^2)^2} [X^I, X^j]^2 \right] .$$

  Thanks to Eduard Balzin for bringing this to my attention.

# 9 Chapter 9

## Section 9.1: Narain Compactifications

- On page 223, the second sentence of the last paragraph of section 9.1 should read:

  The two Majorana-Weyl gravitini and fermions give rise to eight $D = 4$ Majorana gravitini and 56 spin-$\frac{1}{2}$ Majorana fermions

  Thanks to E. Balzin for bringing this to my attention.

## Section 9.2: World-sheet versus space-time supersymmetry

- Equation (9.2.4) should read

$$S_\alpha(z)C_{\dot{\alpha}}(w) = \frac{1}{\sqrt{2}}\sigma^\mu_{\alpha\dot{\alpha}}\ \psi_\mu(w) + \mathcal{O}(z-w)$$

  Thanks to E. Balzin for bringing this to my attention.

- The sentence below equation (9.2.8) should read

  BRST invariance of the fermion vertex implies that the OPE $(e^{-\phi/2}S_\alpha\Sigma^I)(e^\phi G)$ has no single pole term.

  Thanks to M. Tsulaia for bringing this to my attention.

## Section 9.7.2: Consequences of SU(3) holonomy

- Equation (9.7.20) should read

$$F_{ij} = \mathrm{tr}(JR_{ij}) = R_{ij\ kl}J^{kl}\ .$$

  Thanks to E. Balzin for bringing this to my attention.

## Section 9.10: $\mathcal{N} = 2_6$ orbifolds of the type II string

- The sentence after equation (9.10.1) should read
  corresponds to type IIB and IIA respectively.

  Thanks to E. Balzin for bringing this to my attention.

## Section 9.16.5: The Mbius strip amplitude

- The title of this section should read
  The Möbius strip amplitude

  Thanks to E. Balzin for bringing this to my attention.

## Section 9.16.1: Open strings in an internal magnetic field

- Equation (9.16.14) should read

$$\psi^4 - \beta_R \psi^5 + (-1)^a (\bar{\psi}^4 + \beta_R \bar{\psi}^5)\big|_{\sigma=\pi} = 0 \quad , \quad \psi^5 + \beta_R \psi^4 + (-1)^a (\bar{\psi}^5 - \beta_R \bar{\psi}^4)\big|_{\sigma=\pi} = 0 \, ,$$

  Thanks to E. Balzin for bringing this to my attention.

- Equation (9.16.18) should read

$$\left( \psi_\pm - \frac{1 \pm i\beta_L}{1 \mp i\beta_L} \bar{\psi}_\pm \right)\Bigg|_{\sigma=0} = 0 \quad , \quad \left( \psi_\pm + (-1)^a \frac{1 \mp i\beta_R}{1 \pm i\beta_R} \bar{\psi}_\pm \right)\Bigg|_{\sigma=\pi} = 0 \, .$$

  Thanks to E. Balzin for bringing this to my attention.

## Section 9.17:Where is the Standard Model

- Equation 9.17.3 should read

$$S_4 = -\frac{1}{4g_I^2}\int \mathrm{Tr}[F_I F_I] \quad , \quad \frac{1}{g_I^2} = \frac{V_6}{4\pi g_s^2} k_I \ ,$$

Thanks to E. Balzin for bringing this to my attention.

# 10 Chapter 10

## Section 10.1: Calculation of heterotic gauge thresholds

- The sentence after equation (10.1.1) and equation (10.1.2) should be changed to

  To simplify the superstring vertex operators in this chapter we will take $p \to -2p$ in the vertex operators of section 7.5. Therefore the gauge boson vertex operators are
  $$V_G^{\mu,a} = (\partial X^\mu + i(p\cdot\psi)\psi^\mu)\ \bar{J}^a\ e^{2ip\cdot X}\,. \tag{10.1.2}$$

  This is done so that they are compatible with those of section 7.5. Moreover the substitution of $p \to 2p$ is made in equations (10.1.4), (10.1.18), (10.3.1), (10.4.4).

  Thanks to E. Balzin for bringing this to my attention

- Equation (10.1.16) should read
  $$Z^{\text{heterotic}}_{D=4} = \frac{1}{\tau_2\eta^2\bar\eta^2}\sum_{a,b=0}^{1}\frac{1}{2}\frac{\vartheta[^a_b]}{\eta}\ C^{\text{int}}[^a_b]$$

  Thanks to M. Tsulaia for bringing this to my attention

- Equation (10.1.20) should read
  $$Z_2^I = \left.\frac{16\pi^2}{g_I^2}\right|_{1-\text{loop}} = \frac{1}{8\pi^2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\frac{1}{\eta^2\bar\eta^2}\sum_{\text{even}}4\pi i\partial_\tau\left(\frac{\vartheta[^a_b]}{\eta}\right)\ \text{Tr}_{\text{int}}\left[Q_I^2 - \frac{k_I}{4\pi\tau_2}\right][^a_b]$$

  Thanks to M. Tsulaia for bringing this to my attention

- Equation (10.1.21) should read
  $$\left.\frac{16\pi^2}{g_I^2}\right|^{\text{IR}}_{1-\text{loop}} = \frac{1}{2}\int_{\mathcal{F}}\frac{d^2\tau}{\tau_2}\ \text{Str}\ \left[Q_I^2\ \left(\frac{1}{12}-s^2\right)\right]$$

- Equation (10.1.22) should read
  $$\left.\frac{16\pi^2}{g_I^2}\right|^{\text{IR}}_{1-\text{loop}} = -b_I\ \log\left(\frac{\mu^2}{M_s^2}\right) + \text{finite}$$

## Section 10.2: On-shell infrared regularization

- Equation (10.2.15) should read

$$\Delta_I = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \left[ \frac{1}{|\eta|^4} \sum_{\text{even}} \frac{i}{2\pi} \partial_\tau \left( \frac{\vartheta[^a_b]}{\eta} \right) \ \mathrm{Tr}_{\text{int}} \left[ Q_I^2 - \frac{k_I}{4\pi\tau_2} \right] [^a_b] - b_I \right]$$

Thanks to M. Tsulaia for bringing this to my attention

# 11 Chapter 11

## Section 11.5: Self-duality of the IIB string

- Equation (11.5.1) should read

$$g'_{\mu\nu} = e^{-\frac{\Phi}{2}}\, g_{\mu\nu} \quad , \quad \Phi' = -\Phi \quad , \quad B'_2 = C_2 \quad , \quad C'_2 = B_2 \quad , \quad C'_4 = C_4 \; .$$

- The sentence between equations (11.5.8) and (11.5.9) should read

  We have two types of (point-like) BPS states in nine dimensions. The first class consists of KK states on the torus with mass $(2\pi)^2|n_1 + n_2\tau|^2/(\tau_2 A)$.

- Equation (11.5.9) should read

$$M_{11}^2 = (m(2\pi R_{11})^2\tau_2 T_{M_2})^2 + \frac{|n_1 + n_2\tau|^2}{R_{11}^2\tau_2^2} + \cdots \quad ,$$

## Section 11.10: Conifold Singularities and Conifold Transitions

- The caption of figure (11.6) should read as follows:

  *(a) The deformed conifold. (b) The conifold. (c) The resolved conifold. In all cases the square at the base represents the sixes of $S^3$ and $S^2$. In (b) the tip of the cone is singular. In (a) it replaced by a finite size $S^3$. In (c) it is replaced by a finite size $S^2$.*

  Thanks to Richard Garavuso for bringing this to my attention.

# 12 Chapter 12

## Section 12.3: Black hole thermodynamics

- Equation (12.3.11) on page 373 should be replaced by:

$$ds^2 \simeq \frac{(r_+ - r_-)^2}{4r^4{}_+} u^2 d\tau^2 + du^2 \ .$$

- The line before equation (12.3.15) on page 373 starting "It is defined ...." and the two equations below should be replaced by

  In the special case relevant to us here were the boundary surface is $r$ =constant the extrinsic curvature simplifies to

$$K_{ab} = \frac{1}{2} n^\mu \partial_\mu h_{ab} \quad , \quad K = h^{ab} K_{ab} \tag{12.3.15}$$

  where $n^\mu$ is the unit normal to the boundary. Taking a sphere at fixed large $r = r_0$ as the boundary we may evaluate the extrinsic curvature of the RN solution using

$$n^\mu = \frac{1}{\sqrt{g_{rr}}} \left( \frac{\partial}{\partial r} \right) = \frac{\delta^\mu{}_r}{\sqrt{g_{rr}}} \tag{12.3.16}$$

- The first line of equation (12.3.18) on page 374 should be replaced by:

$$\frac{1}{8\pi G} \int_{\partial M} \sqrt{h} K = \beta r^2 \sqrt{f} \frac{K}{2G} \bigg|_{r=r_0} = \beta \frac{r}{4G} (r f' + 4f) \bigg|_{r=r_0} =$$

# 13 Chapter 13

## Section 13.1: Large N gauge theories and string theory.

- Equation (13.1.5) should read

$$\langle X_i^{a\bar{b}} X_j^{c\bar{d}} \rangle \sim \left( \delta^{a\bar{d}} \delta^{b\bar{c}} - \frac{1}{\mathrm{N}} \delta^{a\bar{b}} \delta^{c\bar{d}} \right) \delta_{ij} \ . \tag{13.0.1}$$

## Section 13.5: Bulk fields and boundary operators.

- Equation (13.5.2) should read

$$\phi(x, u)|_{u=0} = \phi_0(x) \ ,$$

- Equation (13.5.4) should read

$$\langle e^{\int d^4x \ \phi_i(x) \ \mathcal{O}^i(x)} \rangle_{CFT_4} = \mathcal{Z}_{\text{string}} \left[ \phi_i(x, u)|_{u=0} = u^{4-\Delta_i} \phi_i(x) \right] \ .$$

## Section 13.8: Correlation functions.

- Equation (13.8.4) should read

$$\phi^i(u, x)\Big|_{u=0} = u^{4-\Delta_i} \phi_0^i(x) \ .$$

## Section 13.8.1: Two-point functions.

- Equation (13.8.5) should read

$$S = \frac{1}{2} \int d^5x \sqrt{g} \left[ (\partial\phi)^2 + m^2\phi^2 \right] = -\frac{1}{2} \int d^5x \sqrt{g} \, \phi(\Box - m^2)\phi + \frac{1}{2} \int d^5x \ \partial_\mu (\sqrt{g} \ \phi \ \partial^\mu \phi) \ .$$

- Equations (13.8.10) to (13.8.14) assumed $L = 1$. To put back $K$ the text should read

$$S_{\text{on-shell}} = -\frac{1}{2}\int d^4\, x\, \sqrt{g} g^{uu}\phi(u,x)\partial_u\phi(u,x)|_{u=0} = \qquad (13.8.10)$$
$$= -\frac{L^3}{2}\int d^4x_1\, d^4x_2\, \phi_0(x_1)\, \phi_0(x_2)\, \int d^4x \frac{K(u,x;x_1)\partial_u K(u,x;x_2)}{u^3}\bigg|_{u=0}\,.$$

Substituting the near-boundary expansion (13.8.8) we obtain

$$\int d^4x \frac{K(u,x;x_1)\partial_u K(u,x;x_2)}{u^3} \simeq \Delta_- u^{2\Delta_- -4}\delta(x_1 - x_2) + \qquad (13.8.11)$$
$$+\frac{4}{C_3}\frac{1}{|x_1 - x_2|^{2\Delta_+}} + \frac{\Delta_+}{C_3^2}u^{2\Delta_+ -4}\int d^4x \frac{1}{|x - x_1|^{2\Delta_+}|x - x_2|^{2\Delta_+}} + \cdots\,.$$

The first term on the right-hand side is a contact term that diverges as we approach the boundary. It must be removed by renormalizing the bulk action. The way to do it is to move the boundary infinitesimally at $u^2 = \epsilon \ll 1$, add a counterterm to remove it and then take the limit $\epsilon \to 0$. The appropriate counterterm is a boundary term:

$$S_{\text{counter}} = -\frac{L^3\, \Delta_-}{2}\epsilon^{\Delta_- -2}\int d^4x\, \phi_0(x)^2 = -\frac{\Delta_-}{2L}\int d^4x\, \sqrt{h^\epsilon}\, \phi(\epsilon, x)^2\,, \quad (13.8.12)$$

where $h_{ij}^\epsilon = \frac{L^2}{\epsilon}\delta_{ij}$ is the metric on the renormalization surface $u^2 = \epsilon$.

The relevant renormalized bulk action (to this order) is

$$S_{ren} = \frac{1}{2}\int_{M_5} d^5x\sqrt{g}\left[(\partial\phi)^2 + m^2\phi^2\right] + \frac{\Delta_-}{2L}\int_{\partial M_5} d^4x\, \sqrt{h}\, \phi^2\,. \quad (13.8.13)$$

With this subtraction, the second term in (13.8.11) will give a finite contribution and all the rest will yield vanishing contributions. We may therefore write, to quadratic order, after a simple rescaling of the sources $\phi_0 \to \frac{\sqrt{C_3}\,\phi_0}{2L^{\frac{3}{2}}}$.

$$S_{ren}^{\text{on-shell}} = -\frac{1}{2}\int d^4x_1\, d^4x_2\, \frac{\phi_0(x_1)\phi_0(x_2)}{|x_1 - x_2|^{2\Delta_+}}\,. \quad (13.8.14)$$

## Section 13.8.2: Three-point functions.

- Equation (13.8.17) should read as

$$S = \frac{1}{2}\int d^5x\sqrt{g}\left[\sum_{i=1}^{3}(\partial\phi_i)^2 + m_i^2\phi_i^2 + 2\xi\phi_1\phi_2\phi_3\right]\,.$$

in order to be compatible with the rest of the equations.

## Exercises for chapter 13

- In exercise 13.46 the third sentence should read:

  Show that as the holographic energy scale U decreases we pass from the perturbative SYM description to the $D_2$ brane supergravity description , to the unwrapped $M_2$ brane supergravity description to the $\mathcal{N} = 8$ SCFT description.

# 14 Chapter 14

## Section 14.1.2: Type IIA $D_0$ Branes and DLCQ.

- Equation (14.1.22) should read

$$v = \frac{1}{\sqrt{1 + 2\frac{R_s^2}{R^2}}} \simeq 1 - \frac{R_s^2}{R^2} + \mathcal{O}(R_s^4)\ .$$

  Thanks to Rene Meyer for bringing this to my attention.

- The third sentence and after before equation (14.1.29) should read

  In that limit, the ten-dimensional Planck scale is given by $M_P^8 \sim R_s/\ell_{11}^9 \to 0$. Although the gravitational interaction seems to become strong, the fact that $g_s \to 0$ and (14.1.28) imply that for the energies in question we can neglect the gravitational/closed string back-reaction to the $\mathrm{D}_0$ branes even if their number N is large.

  Thanks to Rene Meyer for bringing this to my attention.

# Appendix A

- On page 503, in equation (A.6) $\bar{z}' \to \bar{f}(\bar{z})$ should be changed to $\bar{z}' = \bar{f}(\bar{z})$

  Thanks to Moritz McGarrie for bringing this to my attention.

# Appendix B

- Equation (B.1) should read

$$A_p = \frac{1}{p!} A_{\nu_1 \nu_2 \cdots \nu_p} dx^{\nu_1} dx^{\nu_2} \cdots dx^{\nu_p} \ .$$

Thanks to Igmar Cedrell Rosas Lopez for bringing this to my attention.

# Appendix D: Torroidal lattice sums

- Equation (D.1) should read

$$S_{p,q} = \frac{1}{4\pi}\int d^2\sigma\sqrt{\det g}g^{ab}G_{\alpha\beta}\partial_a X^\alpha\partial_b X^\beta + \frac{1}{4\pi}\int d^2\sigma\epsilon^{ab}B_{\alpha\beta}\partial_a X^\alpha\partial_b X^\beta +$$

$$+\frac{1}{4\pi}\int d^2\sigma\sqrt{\det g}\,\sum_I\,\psi^I[\bar\nabla + Y^I_\alpha(\bar\nabla X^\alpha)]\bar\psi^I\,,$$

Thanks to E. Balzin for bringing this to my attention.

# Appendix E: Toroidal Kaluza-Klein reduction

- Equation (E.11) should read

$$Y^I_\alpha = \hat{A}^I_\alpha \quad , \quad A^I_\mu = \hat{A}^I_\mu - Y^I_\alpha A^\alpha{}_\mu \quad , \quad \tilde{F}^I_{\mu\nu} = F^I_{\mu\nu} + Y^I_\alpha F^{A,\alpha}_{\mu\nu}$$

  Thanks to E. Balzin for bringing this to my attention.

- The sentence after equation(E.14) should read
  ...we obtain from (E.7)....

  Thanks to E. Balzin for bringing this to my attention.

- The sentence after equation(E.18) should read
  ...we obtain from (E.8)....

  Thanks to E. Balzin for bringing this to my attention.

# Appendix F

- Equation (F.2) should read

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\left[F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right] \quad , \quad \nabla^{\mu}F_{\mu\nu} = 0 \; .$$

Thanks to Eduard Balzin for bringing this to my attention.

# Appendix H

## Section H.3: Type IIB Supergravity

- Equation (H.22) should read

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}\frac{\partial S\partial\bar{S}}{S_2^2} - \frac{1}{12}|G_3|^2 - \frac{1}{2\cdot 5!}F_5^2\right] + \qquad (H.22)$$
$$+\frac{1}{2i\kappa^2}\int C_4\wedge G_3\wedge\bar{G}_3\ ,$$

Thanks to Karol Plaka for bringing this to my attention.

# Appendix I

## Section I.2: $\mathcal{N} = 2_4$ Supergravity

- In the last sentence of the paragraph above equation (I.13) on page 535, the homogeneity relation $Z^I\ \mathcal{F}_I = 2$ should be replaced by:

$$Z^I\ \mathcal{F}_I = 2\mathcal{F}$$

# Appendix K

## Section K.1: The Minkowski signature AdS

- Equation (K.3) should read

$$X_0 = L\cosh\rho\cos\tau \quad , \quad X_{p+2} = L\cosh\rho\sin\tau \ ,$$

- The sentence "There is another set of coordinates $(u, t, \vec{x})$ with $ru > 0$, $\vec{x} \in \mathbb{R}^p$ that are useful." a bit above equation (K.9) should be changed to :

  There is another set of coordinates $(u, t, \vec{x})$ with $u > 0$, $\vec{x} \in \mathbb{R}^p$ that are useful.

## Section K.4: Fields in AdS

- There is an error in (K.30), and the value of $\xi = 1/6$ quoted below it, is only correct for $p = 2$. We give below the text from the beginning of section K.4 till before equation (K.31) where we have added several new equations and clarifications:

  We will first consider a massive scalar field in $AdS_{p+2}$ with action

$$S = \frac{1}{2}\int d^{p+2}x\sqrt{g}\left[(\partial\phi)^2 + m^2\phi^2\right] \tag{14.0.1}$$

from which the (free) equation of motion follows

$$(\Box - m^2)\phi = 0 \ . \tag{14.0.2}$$

This can be solved by standard methods. In global coordinates, equation (K.7), the solution with well-defined "energy" $\omega$ is of the form

$$\phi = e^{i\omega\tau}\ F(\theta)\ Y_\ell(\Omega_p) \ , \tag{14.0.3}$$

with $Y_\ell(\Omega_p)$ the spherical harmonic on $S^p$, an eigenstate of the Laplacian on $S^p$ with eigenvalue $\ell(\ell + p - 1)$ and

$$F(\theta) = (\sin\theta)^\ell(\cos\theta)^{\Delta_\pm}\ {}_2F_1(a, b, c; \sin^2\theta) \ , \tag{14.0.4}$$

$$a = \frac{1}{2}(\ell + \Delta_\pm - \omega L) \quad , \quad b = \frac{1}{2}(\ell + \Delta_\pm + \omega L) \quad , \quad c = \ell + \frac{1}{2}(p+1) \ , \tag{14.0.5}$$

and

$$\Delta_\pm = \frac{1}{2}(p+1) \pm \frac{1}{2}\sqrt{(p+1)^2 + 4m^2L^2} \ . \tag{14.0.6}$$

To discuss the energy momentum tensor we must be more careful about the conformal invariance of the action in 14.0.1. The action as it stands, even when $m = 0$, is not invariant under general conformal transformations $g_{\mu\nu} \to e^{\rho} g_{\mu\nu}$, $\phi \to e^{-\frac{p}{4}\rho}\phi$. For this to be true, we must add an extra term the couples the scalar to the background curvature

$$S = \frac{1}{2}\int d^{p+2}x\sqrt{g}\left[(\partial\phi)^2 + \left(m^2 + \frac{p}{4(p+1)}R\right)\phi^2\right] \tag{14.0.7}$$

For an AdS background the new term is just a redefinition of the mass, as the curvature in AdS is constant. The new equation of motion is

$$\Box\phi = \left(m^2 + \frac{p}{4(p+1)}R\right)\phi = \left(m^2 - \frac{p(p+2)}{4L^2}\right)\phi \tag{14.0.8}$$

where in the last equality we used the value of the constant curvature of $AdS_{p+2}$.

The energy momentum tensor is given by

$$T_{\mu\nu} \equiv \frac{\delta S}{\sqrt{g}\delta g^{\mu\nu}} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}g_{\mu\nu}((\partial\phi)^2 + m^2\phi^2) + \tag{14.0.9}$$

$$+\frac{p}{8(p+1)}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + R_{\mu\nu} - \frac{R}{2}g_{\mu\nu})\phi^2 \; ,$$

the last term coming from the coupling of the scalar to the scalar curvature of the background. Using the equations of motion we may rewrite the stress tensor in the form

$$T_{\mu\nu} = \frac{1}{4(p+1)}\left[(p+2)\partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}(\partial\phi)^2\right] - \frac{p}{4(p+1)}\phi\nabla_\mu\nabla_\nu\phi + \tag{14.0.10}$$

$$+g_{\mu\nu}\left[-\frac{m^2}{4(p+1)} + \frac{p^2}{16(p+1)^2(p+2)}R\right]\phi^2$$

The trace is given by

$$T_\mu{}^\mu = -\frac{m^2}{2}\phi^2 \tag{14.0.11}$$

where we used once more the equations of motion (14.0.7). It vanishes in the massless case.

- The sentence below equation K.34 should read:

  "This is possible only when $\Delta_\pm$ is real".

- The second sentence below equation K.40 should read:

  "It is the one that corresponds to the normalizable mode in Minkowski signature and also to the $\Delta_-$ solution in (K.27)".

- The next sentence should read:

  "Note that, in contrast to the global case, the spectrum is now continuous."

- Equation (K.41) should read

$$\phi(u,x)|_{u=0} = u^{p+1-\Delta}\phi_0(x) \; ,$$

- Equation (K.42) should read

$$(\Box - m^2)K(u,x;x') = 0 \quad , \quad K(u,x;x')|_{u=0} = u^{p+1-\Delta} \;\; \delta^{(p+1)}(x-x') \; .$$

- Equation (K.43) should read

$$K(u,q)|_{u=0} = u^{p+1-\Delta} \; .$$

- The sentence above equation (K.44) and equation (K.44) should be changed to:

  As we argued in section 13.5 on page 429 we must impose the condition very close to the boundary at $u=\epsilon$, namely $K(\epsilon,q) = \epsilon^{p+1-\Delta}$. We find

$$K_\epsilon(u,x-x') = \frac{(u\epsilon)^{\frac{p+1}{2}}}{\epsilon^\Delta} \int \frac{d^{p+1}q}{(2\pi)^{p+1}} \frac{K_\nu(\sqrt{q^2}\; u)}{K_\nu(\sqrt{q^2}\; \epsilon)} \; e^{ip\cdot(x-x')} \; .$$

- The text after equation (K.44) and till the end of the subsection is rewritten below to make it more clear.

  We may alternatively construct the propagator in configuration space. We will rotate to Euclidean space for convenience. You are asked in exercise **13.18** on page 481 to verify that the function

$$f(u,x;x') = \frac{u^\Delta}{(u^2+|x-x'|^2)^\Delta} \quad , \quad \Delta(\Delta-p-1) = m^2L^2 \; ,$$

  satisfies the massive Laplace equation everywhere.

  Moreover, if $x \neq x'$, $f$ vanishes as $u \to 0$, while for $x = x'$, it diverges at $u=0$. We may compute

$$\int d^{p+1}x \; f(u,x;x') = u^{p+1-\Delta}\Omega_p \int_0^\infty \frac{\zeta^p d\zeta}{(1+\zeta^2)^\Delta} = C_p \; u^{p+1-\Delta}$$

with

$$C_p = \frac{\Gamma\left[\frac{p+1}{2}\right]\Gamma\left[\Delta-\frac{p+1}{2}\right]\Omega_p}{2\Gamma[\Delta]} = \pi^{\frac{p+1}{2}}\frac{\Gamma\left[\Delta-\frac{p+1}{2}\right]}{\Gamma\left[\Delta\right]}\ ,$$

where are usual $\Omega_p$ is the volume of the unit $S^p$ as in (8.8.11 on page 217). This implies that

$$\lim_{u\to 0}\frac{u^\Delta}{(u^2+|x-x'|^2)^\Delta} = C_p\ u^{p+1-\Delta}\ \delta^{(p+1)}(x-x')\ ,$$

Therefore, the normalized bulk-to-boundary propagator is

$$K_\Delta(u,x;x') = \frac{\Gamma\left[\Delta\right]}{\pi^{\frac{p+1}{2}}\Gamma\left[\Delta-\frac{p+1}{2}\right]}\frac{u^\Delta}{(u^2+|x-x'|^2)^\Delta}\ ,$$

To have the correct asymptotics we must generically choose $\Delta=\Delta_+$, the larger of the two roots. Then the solution of the massive Laplace equation can be written as

$$\phi(u,x) = C_p^{-1}\int d^{p+1}x'\ \frac{u^{\Delta_+}}{(u^2+|x-x'|^2)^{\Delta_+}}\phi_0(x')\quad ,\quad \lim_{u\to 0}\ \phi(u,x)\simeq u^{\Delta_-}\phi_0(x)+\cdots\ ,$$

and asymptotes properly at the boundary, proportional to the leading solution. This justifies our choice of branch.

- Appendix K.4.3 on the bulk to bulk propagator is rewritten to be valid for general dimension and to correct a few missprints:

  The propagator is defined as usual as the inverse of the kinetic operator

$$(\Box - m^2)G(u,x;u',x') = \frac{u^{p+2}}{L^{p+2}}\delta(u-u')\delta^{(p+1)}(x-x')\ , \qquad (14.0.12)$$

  where we are using Poincaré coordinates. With this normalization the solution of the equation

$$(\Box - m^2)\phi = J \qquad (14.0.13)$$

  is solved by

$$\phi(u,x) = \int du' d^{p+1}x' \sqrt{g}\ G(u,x;u',x')\ J(u',x') \qquad (14.0.14)$$

  There is however an issue of boundary conditions, at the boundary of $\mathrm{AdS}_{p+2}$ that we will return to below.

  To construct $G$ we will use the invariant $\mathrm{AdS}_{p+2}$ distance

$$\eta^2 = \frac{u^2+u'^2+(x-x')^2}{2uu'}\ . \qquad (14.0.15)$$

The two linearly independent solutions properly normalized according to (14.0.12) are

$$G_{\Delta_+}(u,x;u',x') = -\frac{\Gamma\left[\Delta_+\right]}{2^{\Delta+1}\pi^{\frac{p+1}{2}}\Gamma\left[\Delta_+ - \frac{p-1}{2}\right]L^p}\,\eta^{-2\Delta_+}\,{}_2F_1\left(\frac{\Delta_+}{2},\frac{\Delta_+ +1}{2};\Delta_+ + \frac{1-p}{2};\frac{1}{\eta^4}\right)\;, \tag{14.0.16}$$

and $G_{\Delta_-}(u,x;u',x')$ where $\Delta_\pm$ are given in (K.40). The linear combination $C\;G_{\Delta_+} + (1-C)G_{\Delta_-}$ is the general solution to (14.0.12).

If we Fourier transform the $x$ coordinates the propagator becomes

$$G_{\Delta_\pm}(u,x;u',x') = -\frac{(uu')^{\frac{p+1}{2}}}{L^p}\int\frac{d^{p+1}q}{(2\pi)^{p+1}}e^{-iq\cdot(x-x')}\begin{cases} I_\nu(qu)K_\nu(qu'), & u<u' \\ I_\nu(qu')K_\nu(qu), & u>u'. \end{cases}$$

where as usual $\nu = \Delta_+ - \Delta_-$.

As one of the bulk points moves to the boundary, the bulk-to-bulk propagator asymptotes to the bulk-to-boundary one

$$\lim_{u\to 0} G_\Delta(u,x;u',x') = \frac{u^\Delta}{p+1-2\Delta}K_\Delta(x;u',x')\;. \tag{14.0.17}$$

For $\Delta > \frac{p+1}{2}$ it is $G_{\Delta_+}$ that vanishes faster at the boundary.

In most applications, calculations are done at the regularized boundary at $u=\epsilon$. It is necessary to define a bulk propagator $G_\epsilon$ that vanishes at the regularized boundary. The form suggested from (14.0.17) is

$$G_\epsilon(u,x;u',x') = G(u,x;u',x') + \frac{(uu')^{\frac{p+1}{2}}}{L^p}\int\frac{d^{p+1}q}{(2\pi)^{p+1}}\,e^{-iq\cdot(x-x')}K_\nu(qu)K_\nu(qu')\frac{I_\nu(q\epsilon)}{K_\nu(q\epsilon)}\;, \tag{14.0.18}$$

$G_e$ satisfies (14.0.12) and

$$G_\epsilon(\epsilon,x;u',x') = 0 \quad , \quad \partial_u G_\epsilon(\epsilon,x;u',x') = -\frac{\epsilon^{\Delta-1}}{L^p}\;K_\epsilon(u',x';x)\;. \tag{14.0.19}$$