Preprint typeset in JHEP style - HYPER VERSION

Non-perturbative Quantum Field Theory

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Abstract: The purpose of the course is to develop several methods that have been used over the years to deal with non-perturbative physics in quantum field theory. Such methods are rarely taught in today's graduate programs. Such methods complement modern approaches to non-perturbative physics using the holographic correspondence. There are two parts. The first deals with non-supersymmetric theories. The second deals with the special properties of supersymmetric theories.

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Contents

This a course spanning 66 academic hours (each hour= 45 minutes).

1. Confinement in Quantum Chromodynamics (12 hours)

1. The string picture of hadrons (1 hour).
2. The Nielsen-Olesen vortex as stable confined magnetic flux (1 hour).
3. Generalities on the realization of global and gauge theories. Elitzur's theorem (1 hour).
4. Wilson loops and a criterion for confinement. Timelike loops, spacelike loops. The Wilson loop as a "gauge" of magnetic flux (1 hours).
5. Confinement in 2+1 YM theories (1 hour).
6. 't Hooft operators and their braiding with Wilson loops (1 hour).
7. Magnetic and electric fluxes in a box and duality relations (2 hours).
8. Confinement of quarks and monopoles (1 hour).
9. Deconfinement at finite temperature and the phase transition (1 hour).
10. Confinement with Dynamical Quarks (1 hour).
11. Dynamical Quarks at finite temperature (1 hour).

2. Chiral symmetry and chiral symmetry breaking (22 hours)

1. The pair-condensate instability (1 hour).
2. The Nambu-Jona-Lasinio model and dynamical chiral symmetry breaking (1 hour).
3. The chiral symmetry at finite temperature (1 hour).
4. Chiral anomalies and their implications (1 hour).
5. The $U(1)_A$ problem (2 hours) .
6. Instantons and the θ parameter (2 hours).
7. The chiral selection rule (2 hours).
8. Anomalies and the realization of chiral symmetry (the 't Hooft anomaly matching conditions) (2 hours).
9. Anomaly matching in QCD. Persistence of mass. Complementarity. (4 hours)
10. Chiral gauge theories.(2 hours)
11. The trace anomaly. (1 hour)
12. The non-perturbative SU(2) (Witten) anomaly. (1 hour)

3. The large N_c limit (12 hours)

1. Generalities. (1 hour)
2. The Gross-Neveu model. (2 hours)
3. The large-N_c limit in QCD and 't Hooft's string expansion. (3 hours)
4. Meson-Glueball phenomenology at large-N_c. (3 hours)
5. θ dependence in large-N_c YM. (1 hour)
6. Baryons in the large-N_c expansion. (2 hours)

4. Lattice gauge theories (8 hours)

1. The Wilson formulation. (2 hours)
2. The strong coupling expansion.(1 hour)
3. The character expansion (1 hour)
4. Z_N gauge theory and Kramers-Wannier duality. (1.5 hours)
5. Wilson loops and 't Hooft loops. (0.5 hours)
6. Phase diagrams of lattice theories. (2 hours)

5. Supersymmetric non-perturbative dynamics (12 hours)

5.1 Supersymmetric gauge theories (4 hours)

1. Introduction and overview of phenomena that will be studied.
2. Supersymmetry multiplets, supersymmetric Lagrangians ($N = 1, 2$)
3. Effective actions, holomorphy and symmetries, non-renormalization theorems.
4. Phases of gauge theories.
5. Exact beta-functions

5.2 N=2 supersymmetric gauge theories (4 hours)

1. The perturbative regime (SU(2)).
2. The exact quantum moduli space.
3. The BPS spectrum.
4. The generalization of the Seiberg-Witten theory to other gauge groups.
5. More on duality, Argyres-Seiberg duality, Gaiotto generalizations.

5.3 N=1 supersymmetric gauge theories (4 hours)

1. Perturbative N=1 Supersymmetric QCD, symmetries and vacua.
2. Non-perturbative sQCD in the various regimes (pure sYM, N$_f$¡N$_c$ no vacuum, N$_f$=N$_c$ confinement with chiral symmetry breaking, N$_f$=N$_c$+1 confinement without chiral symmetry breaking, N$_f$ $> N_c$+1)
3. The 't Hooft anomaly matching conditions in sQCD.

4. Seiberg duality: formulation, implications, checks.
5. Other examples of Seiberg duality
6. a-maximization: towards an a-theorem.
7. Example: adjoint-SQCD

6. On the bibliography

The main parts of sections 1-4 will be based on a combination of the old material of Preskill's lectures, [1] as well as from various other lecture notes and books like:

1. Coleman's Erice Lectures, [2]. This has relevant chapters well explained for beginners for solitons, instantons and large N_c,
2. Frishman's and Sonennschein's book [3]. It contains for our purposes, Instantons in QCD, magnetic monopoles and (baryon) solitons in the Skyrme model, Large N_c methods in QCD. It also contains relevant two dimensional physics namely, the 't Hooft solution of two dimensional QCD, and the physics of baryons.
3. The standard review on basic lattice methods [4]
4. 't Hooft's classic review on instantons and the $\mathrm{U}(1)_A$ problem, [5].
5. 't Hooft's classic review on Monopoles, Instantons and Confinement, [6].
6. Manohar's lectures on Large-N_c QCD, [7].

The lectures on non-perturbative methods in supersymmetric theories will be based on mainly references [8] and [9]. Further references in this directions with different emphasis are given in [10]-[16].

References

Preskill [1] John Preskill's lecture notes available from his web page at Caltech, http://www.theory.caltech.edu/∼preskill/notes.html#qcd

Coleman [2] S. Coleman, *"Aspects of symmetry"*, the Erice lectures. Table of contents

Cobi [3] Y. Frishman and J. Sonnenschein, *"Non-Perturbative Field Theory*: From Two Dimensional Conformal Field Theory to QCD in Four Dimensions", Cambridge University Press, Table of contents

Itzykson [4] J. M. Drouffe and C. Itzykson, *"Lattice gauge fields"*, Physics Reports **38** (1978) 133-175.

tHooft1 [5] G. 't Hooft, *"How instantons solve the U(1)$_A$ problem"*, Physics Reports **142**, (1986) 357-387.

tHooft2 [6] G. 't Hooft, *"Monopoles, Instantons and Confinement"*, Lecture notes.

Manohar [7] A. Manohar, "Large-N QCD", Les Houches Lecture notes.

lerche [8] W. Lerche, *"Introduction to Seiberg-Witten theory and its stringy origin"*, http://arxiv.org/abs/hep-th/9611190

argyres [9] P. Argyres, *"Introduction to global supersymmetry"*, Lecture Notes

seiberg [10] , K. Intriligator, N. Seiberg, *"Lectures on supersymmetric gauge theories and electric-magnetic duality"*, http://arxiv.org/abs/hep-th/9509066

flume [11] R. Flume, L. O'Raifeartaigh, I. Sachs, *"Brief resume of Seiberg-Witten theory"*,http://arxiv.org/abs/hep-th/9611118

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peskin [13] M. Peskin, *"Duality in supersymmetric Yang-Mills theory"*, http://arxiv.org/abs/hep-th/9702094

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Wipf [15] A. Wipf, "*Non-perturbative methods in supersymmetric theories*", hep-th/0504180.

strassler1 [16] M. Strassler, *"An Unorthodox Introduction to Supersymmetric Gauge Theory"*, http://arxiv.org/abs/hep-th/0309149.