"Quantum Chaos and Random Matrix Theories"

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Abstract

I shall present the fundamentals of quantum chaos, especially the statistical properties of energy spectra of the stationary Schrödinger equation. For Hamilton systems having the classical limit, we find in the semiclassical limit, that the spectral fluctuations obey the Poissonian statistics, whilst for classically ergodic and chaotic systems the statistics of the eigenvalues of the Gaussian random matrices applies. In both cases we have universality, as no free parameter appears. If the system is of the mixed type (generic system), having divided classical phase space (regular motion on invariant tori and chaotic motion on complementary initial conditions) we find in the strict semiclassical limit the statistical independence of regular and chaotic energy levels (Berry and Robnik 1984). This BR-picture rests upon the principle of uniform semiclassical condensation of Wigner functions (WF) of eigenstates, where we see in the semiclassical limit that WF fill uniformly the classically accessible phase space (regular WF on invariant tori, chaotic ones on the chaotic components), and the eigenvalues do not interact. At lower energies or larger values of the effective Planck constant we observe new effects, namely in the first place localization of chaotic eigenstates (nonuniform spreading of WF) and interaction of eigenvalues (due to the tunneling, i.e. overlap of semiclassical WF in classically forbidden regions). I shall present most recent models of random matrices which excellently describe these effects. Important examples are e.g. various 2D billiard systems and the hydrogen atom in strong magnetic field.