ABSTRACT
Recently much attention has been paid to Bloch dynamics of a Bose-Einstein condensate (BEC) loaded into optical lattices and subject to a static, for example, gravitational field [1,2]. We thoroughly compare two theoretical approaches to this problem – the mean-field description, based on the discrete nonlinear Schroedinger equation (DNLSE), and the microscopic description, based on the Bose-Hubbard model. Within the mean-field approach the main phenomena related to the Bloch dynamics are the dynamical instability (also known as modulation instability) and self-thermalization due to the onset of classical chaos in the DNLSE. It is argued that the quantum manifestations of these phenomena are the depletion of Floquet-Bogoliubov states, defined as the "low-energy" eigenstates of the evolution operator over one Bloch period, and the decoherence of the BEC. We also show that for the considered problem the correspondence between the mean-field and microscopic descriptions depends not only the number of particles but also on the magnitude and direction (for 2D or 3D lattices) of the static field.