University of Crete
Department of Physics

Physics Colloquium

Thursday, 11 April 2024 | 17:00 – 18:00, Seminar Room, 3rd floor

Botzmann-Gibbs distributions beyond statistical physics: Gaussian processes, spatiotemporal statistics and machine learning

Dionissios T. Hristopulos

School of Electrical and Computer Engineering, Technical University of Crete, Chania, Greece

ABSTRACT

Gaussian process regression is a popular method in machine learning which can be used to reconstruct functions in potentially high-dimensional spaces based on sparse (i.e., scattered) data. The main engine of Gaussian processes are covariance kernels. For geo-referenced data, the covariance kernel enforces correlations between different spatial locations (and/or times). In Boltzmann-Gibbs (BG) models of spatial processes, the dependence is enforced by means of interactions between the values of the field at neighboring sites and/or times. The interactions are controlled by respective parameters and contribute to a scalar energy term which provides the exponent of the exponential BG probability density. An advantage of the BG representation is that it can lead to sparse interaction patterns in space and time. Hence, it is no surprise that the magnetic Ising model inspired research in spatial statistics. In contrast with Gaussian processes, the covariance functions of BG models are not known a priori but are calculated in terms of the parameters that control the local interactions. For Gaussian BG models, explicit expressions for the covariance kernels can be derived in certain cases using a continuum formulation. In the discrete case, the spatiotemporal structure of the interactions determines the model's precision matrix (inverse covariance matrix). The latter is sparse by construction if the interactions are local. This property can lead to efficient interpolation algorithms with considerable computational gains compared to standard approaches.

This presentation reviews recent progress in the application of Boltzmann-Gibbs models to spatial and space-time data. The topics that will be discussed include: (1) The construction of new covariance models based on continuum-space BG models. (2) The connection of lattice-based BG models to Gauss-Markov random fields and stochastic partial differential equations. (3) The extension of BG models to scattered spatiotemporal data by means of interactions modulated via kernel functions. It will be shown, using different case studies, that BG models provide a flexible framework for the development of space-time data models which are computationally efficient and therefore useful for big datasets. In addition, while testing a model's permissibility is not an easy task in the case of multivariate, covariance-based models, certain BG models can lead to straightforward permissibility conditions.